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Signal modeling for two-dimensional image structures

Di Zang [*,1], Gerald Sommer [2]

Cognitive Systems Group, Department of Computer Science, Christian Albrechts University of Kiel, 24118 Kiel, Germany

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Abstract

This paper presents a novel approach towards two-dimensional (2D) image structures modeling. To obtain more degrees of freedom, a 2D image signal is embedded into a certain geometric algebra. Coupling methods of differential geometry, tensor algebra, monogenic signal and quadrature filter, a general model for 2D image structures can be obtained as the monogenic extension of a curvature tensor. Based on this model, local representations for the intrinsically one-dimensional (i1D) and intrinsically two-dimensional (i2D) image structures are derived as the monogenic signal and the generalized monogenic curvature signal. From the local representation, independent features of local amplitude, phase and orientation are simultaneously extracted. Compared with the other related work, the remarkable advantage of our approach lies in the rotationally invariant phase evaluation of 2D structures, which delivers access to phase-based processing in many computer vision tasks.

Keywords: Monogenic curvature tensor; Generalized monogenic curvature signal; Phase; Signal modeling

1. Introduction

Model-based image representation plays an important role in many computer vision tasks such as object recognition, motion estimation, image retrieval, etc. Therefore, signal modeling for local structures is of high significance in image processing. There are bulk of researches for intensity-based modeling, see [1–5]. However, those approaches suffer from illumination variations. Therefore, that intensively investigated area of research is not adequate to model local structures. On the other hand, phase information carries most essential structure information of the original signal [6]. It is invariant with respect to illumination changes. Consequently, modeling of local structures should take both the intensity and phase information into consideration.

In one-dimensional (1D) signal processing, the analytic signal [7] is an important complex-valued model which can be used for speech recognition, seismic data analysis, airfoil design and so on. The polar representation of the analytic signal yields the local amplitude and local phase, which are measures of quantitative and qualitative information of a signal, respectively. In 1D case, there exist four types of structures. They are the peak, pit, decreasing slope and increasing slope. The local amplitude is invariant with respect to local structures and it indicates the energetic information of the signal. The local phase allows to distinguish structures and it is invariant with respect to the local amplitude. If the local structure varies, the local phase will correspondingly change. Local amplitude and local phase are independent of each other and they fulfill the properties of invariance and equivariance. Invariance means that a feature value is not changed by a certain group acting on a signal. Opposite to invariance, equivariance means there is a monotonic dependency of a feature value on the parameter of the group action. If a set of features includes only invariant and equivariant features, it thus has the property of invariance–equivariance. In addition to satisfying the requirement of invariance and equivariance, if a set

* Corresponding author. Fax: +49 431 8807550.
E-mail addresses: zd@ks.informatik.uni-kiel.de (D. Zang), gs@ks.informatik.uni-kiel.de (G. Sommer).
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of features is at the same time a unique description of the signal, it then performs a split of identity [8]. The split of identity indicates that different features represent mutually different properties of the signal and the whole set of features describes completely the signal. Hence, the analytic signal performs a split of identity.

In 2D images, there exist infinite many types of structures. These can be classified with different features such as their intrinsic dimensions, the number and shape of junctions, or the type of curvature in a differential geometric setting. According to their intrinsic dimensionality, 2D images can locally belong to the intrinsically zero dimensional (i0D) signals which are constant signals, intrinsically one dimensional (i1D) signals representing straight lines and edges and intrinsically two-dimensional (i2D) signals which do not belong to the above two cases. The i2D structures are composed of curved edges and lines, junctions, corners and line ends, etc. Intrinsic dimensionality [9] is a local property of a multidimensional signal, which expresses the number of degrees of freedom necessary to describe local structure. The term intrinsic dimension used in image processing corresponds to the term codimension in mathematics. In [9], a discrete concept of the intrinsic dimensionality has been proposed and it was later extended to a continuous one by Krüger and Felsberg [10]. The i1D and i2D structures carry most of the important information of the image, therefore, correct characterization of them has great significance for many computer vision applications.

Many approaches have been proposed for the signal representation of local image structures. Tensors turn out to be interesting data structures for image analysis. The structure tensor [1] and the energy tensor [11] estimate the main orientation and the energy of the i2D signal. However, the split of identity is lost, because the phase is neglected. In [3], a nonlinear image operator for the detection of locally i2D signals was proposed, but it captures no information about the phase. There are lots of papers concerned with applications of the analytic signal for image analysis. But they have serious problems in transferring that concept from 1D to 2D in a rotation-invariant way. One of the most interesting proposals is the boundary tensor [2]. Recently, it has been identified as a kind of quadrature operator [12], which applies a tensor representation of the Riesz transform as a generalization of the Hilbert transform. The partial Hilbert transform and the total Hilbert transform [13] provide some representations of the phase in 2D. Unfortunately, they lack the property of rotation invariance and are not adequate for detecting i2D features. Bülow and Sommer [14] proposed the quaternionic analytic signal, which enables the evaluation of the i2D signal phase, however, this approach also has the drawback of being not rotationally invariant. For i1D signals, Felsberg and Sommer [15] proposed the monogenic signal as a novel model. It is a rotationally invariant generalization of the analytic signal in 2D and higher dimensions. In that work, the application of the Riesz transform has been proposed as generalized Hilbert transform in image analysis. In [16], the 3D monogenic signal has been used for image sequence analysis. From the monogenic signal, the local amplitude and a local phase representation can be simultaneously extracted. The monogenic signal delivers an orthogonal decomposition of the original signal into amplitude, phase and orientation. Thus, the monogenic signal has the property of split of identity [15]. However, it captures no information of the i2D part. A 2D phase model was proposed in [17], where the i2D signal is split into two perpendicularly superposed i1D signals and the corresponding two phases are evaluated. The operator derived from that signal model takes advantage of spherical harmonics up to order three. It delivers a new description of i2D structure by a so-called structure multivector. Unfortunately, steering is needed and only i2D patterns superimposed by two perpendicular i1D signals can be correctly handled.

Quite another approach of local signal analysis is based on differential geometry of curves and surfaces [18,19]. The main points of concern are some invariance properties of signal analysis and regional symmetry with respect to certain combinations of Gaussian and mean curvatures of local surface patterns in a Gaussian multi-scale framework [20,21]. We will pick up the differential geometry model of surfaces. But instead of a Gaussian blurring operator, we will apply a Poisson blurring operator as a consequence of the algebraic embedding we use.

Our purpose is to build a general model for all 2D structures without necessarily delivering all parameters for describing the local structure. This model should contain both the amplitude and phase information of 2D structures in a rotation-invariant manner. In other words, the new model should be an extension of the analytic signal to the 2D case. In this paper, we present a novel signal model which covers 2D structures of all intrinsic dimensionalities. By embedding our problem into a certain geometric algebra, more degrees of freedom can be obtained to derive a complete representation for the 2D structure. Based on the differential geometry, we are able to design that general model for 2D structures in a rotation-invariant manner by coupling the methods of tensor algebra, monogenic signal and quadrature filter. The proposed model can be considered as the monogenic extension of a curvature tensor. From this model, a local signal representation for i1D structures is obtained. It is exactly the monogenic signal [15] as a special case of this general model. The local representation for i2D structures, referred as the generalized monogenic curvature signal, can also be derived based on the proposed model.

From the generalized monogenic curvature signal, three independent local features can be extracted. They are the amplitude, phase and orientation just like in the case of the monogenic signal. Hence, the generalized monogenic curvature signal also performs the split of identity, i.e., the invariance–equivariance property of signal decomposition. The energy output (square of the amplitude) can be