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Fast computation of Jacobi-Fourier moments for invariant image recognition

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ABSTRACT

The Jacobi-Fourier moments (JFMs) provide a wide class of orthogonal rotation invariant moments (ORIMs) which are useful for many image processing, pattern recognition and computer vision applications. They, however, suffer from high time complexity and numerical instability at high orders of moment. In this paper, a fast method based on the recursive computation of radial kernel function of JFMs is proposed which not only reduces time complexity but also improves their numerical stability.

1. Introduction

Jacobi-Fourier moments (JFMs) are a generic class of orthogonal rotation invariant moments (ORIMs) recently introduced by Ping et al. [1] and further investigated by Hoang and Tabbone [2]. They provide a wide range of ORIMs whose radial kernel functions are polynomials. The widely used Zernike moments (ZMs) [3], pseudo-Zernike moments (PZMs) [4], orthogonal Fourier-Mellin moments (OFMMs) [5], and Chebyshev Harmonic Fourier moments (CHFMs) [6] are special cases of JFMs. First introduced by Teague [3] to image analysis, ZMs have found several applications in image processing, pattern analysis and computer vision. The PZMs were introduced by Bhatia and Wolf [4] and these have been observed to be more resilient to image noise than ZMs. The OFMMs were introduced by Sheng and Shen [5] which are useful for describing small images. Ping et al. [6] introduced CHFMs and observed it to possess better noise sensitivity and reconstruction capability as compared to OFMMs. A comparative performance analysis carried out by Teh and Chin [7] reveals that ZMs have superior image description capabilities than other orthogonal and non-orthogonal moments. Recently, Singh and Upneja [8] have carried out performance analysis of the three ORIMs and observed that ZMs outperform PZMs and OFMMs in several ways. The generalisation of these moments and the availability of a wide range of ORIMs in the form of JFMs have opened several avenues for their applications in many image processing and pattern recognition problems.

The kernel functions of the JFMs are orthogonal and complete. The property of orthogonality provides minimum information redundancy which enables to describe an image uniquely with a few moment coefficients. The magnitudes of these moments are rotation invariant. They are computed on a unit disk. When applied on an image of any resolution, they can be made scale invariant after performing mapping transformation [8,9]. If the centre of the unit disk is taken at the centre of the mass of an image, they become translation invariant. Since they are computed through an integration process, they provide excellent noise resilience. These moments are non-discrete and hence they provide infinite number of moments unlike their discrete counterparts. The low order moments describe the overall aspects of an image while the high order moments provide fine details.

Despite several useful characteristics of JFMs, they suffer from high computation complexity. The radial kernel functions of JFMs are polynomials with degree equal to the order of polynomials. If JFMs of all orders upto a maximum order n_{max} are computed, then the order of time complexity is $O(N^2 n_{max}^2)$ for an image of size $N \times N$. This order is very high when n_{max} is large. A large number of moments are required for many applications, such as in image watermarking [10,11] and due to the high time complexity their usefulness for such applications is prohibitive. There are a few fast algorithms for ZMs, PZMs and OFMMs [12–15] which reduce the time complexity of these three JFMs. There is no such fast algorithm for the generic moments based on JFMs. In this paper, we present a fast algorithm which reduces the time complexity of the JFMs computation from $O(N^2 n_{max}^2)$ to $O(N^2 n_{max})$. The fast algorithm is based on the recursive computation of the radial

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and angular kernel functions of the moments. The proposed recursive method not only reduces the time complexity of moment computation but also enhances numerical stability of high order moments which is reflected in the lower values of image reconstruction error.

The rest of the paper is organised as follows. An overview of JFMs and its computational framework for digital images are presented in Section 2. The proposed fast recursive algorithm for the computation of radial polynomials of JFMs is given in Section 3. In order to provide an overview of the fast computation of JFMs, a brief description of the existing fast computation of its angular functions and 8-way symmetry/anti-symmetry property are described in Section 4. Also, in this section a theoretical comparison with the fast computation of a recently developed orthogonal rotation invariant global image descriptor, generic polar harmonic transforms (GPHTs) is carried out. The effect of the proposed recursive algorithm for the fast computation of radial functions on its enhanced numerical stability is presented in Section 5. Detail experiments are conducted in Section 6 analysing the time complexity and numerical stability. Section 7 concludes the paper.

2. The Jacobi-Fourier moments (JFMs)

The JFMs of order n and repetition m for an image function $f(r,\theta)$ on a unit disk in polar coordinates are defined by [1,2]

$$H_{nm} = \int_0^{2\pi}\int_0^1 f(r,\theta)V^*_{nm}(p,q,r,\theta) r dr\, d\theta, \tag{1}$$

where p and q are real parameters, n is a non-negative integer, m is an integer and $V^*_{nm}(p,q,r,\theta)$ is the complex conjugate of JFMs kernel function $V_{nm}(p,q,r,\theta)$ which is separable in radial and angular functions.

$$V_{nm}(p,q,r,\theta) = R_n(p,q,r)w_m(\theta). \tag{2}$$

The function $R_n(p,q,r)$ is the deformed Jacobi polynomial which is expressed in the form of Jacobi polynomial as follows:

$$R_n(p,q,r) = b_n(p,q,r)J_n(p,q,r), \tag{3}$$

where $J_n(p,q,r)$ are shifted Jacobi polynomials and $b_n(p,q,r)$ are weight functions used to make the radial functions orthonormal on the unit disk.

$$J_n(p,q,r) = \frac{n!(q-1)!}{(p+n-1)!}\sum_{k=0}^{n}\frac{(-1)^k(n+k+p-1)!r^k}{k!(q+k-1)!(n-k)!}, \tag{4}$$

$$b_n(p,q,r) = \left[\frac{(q+n-1)!(p+n-1)!(p+2n)}{n![(q-1)!]^2(p-q+n)!}(1-r)^{p-q}r^{q-2}\right]^{1/2}, \tag{5}$$

The function $w_m(\theta)$ is orthonormal in the circular domain $[0, 2\pi]$.

$$w_m(\theta) = \frac{1}{\sqrt{2\pi}}e^{jm\theta}. \tag{6}$$

In order to facilitate the computation process the radial functions $R_n(p,q,r)$ are rewritten as:

$$R_n(p,q,r) = [(p+2n)(1-r)^{p-q}r^{q-2}]^{1/2}A_n(p,q)P_n(p,q,r), \tag{7}$$

where

$$A_n(p,q) = \left[\frac{n!(q+n-1)!}{(p+n-1)!(p-q+n)!}\right]^{1/2}, \tag{8}$$

and

$$P_n(p,q,r) = \sum_{k=0}^{n}\frac{(-1)^k(n+k+p-1)!r^k}{k!(q+k-1)!(n-k)!}. \tag{9}$$

The kernel functions $V_{nm}(p,q,r,\theta)$ are orthogonal on a unit disk [1,2]

$$\int_0^{2\pi}\int_0^1 V_{nm}(p,q,r,\theta)V^*_{n'm'}(p,q,r,\theta)\, r dr\, d\theta = \delta_{nn'}\delta_{mm'}. \tag{10}$$

where δ_{ij} is the kronecker delta.

The ZMs, PZMs, OFMMs and CHFMs are special cases of JFMs [1,2]:

i) ZMs : $p=|m|+1, q=|m|+1$.

ii) PZMs : $p=2|m|+2, q=2|m|+2$.

iii) OFMMs : $p=2, q=2$.

iv) CHFMs : $p=2, q=3/2$. (11)

It is noted that the factorial terms involving the parameter $q=3/2$ for CHFMs is converted into gamma function using the relationship $q! = \Gamma(q+1)$ which can be evaluated recursively using $\Gamma(q) = (q-1)\Gamma(q-1)$ with $\Gamma(1/2) = \sqrt{\pi}$.

2.1. JFMs for digital Images

A digital image $f(x,y)$ is defined over a rectangular grid $M \times N$ with M and N representing rows and columns, respectively. For simplicity, we consider a square image $N \times N$. Let (i,k) represent a pixel. Then the following geometric transformation performs a mapping process which maps a square image into a unit circle [8,9].

$$x_i = \frac{2i+1-N}{D}, \quad y_k = \frac{2k+1-N}{D}, \quad i,k = 0,\ 1,\ \ldots,\ N-1, \tag{12}$$

where

$$D = \begin{cases} N\sqrt{2}, & for\ outer\ unit\ disk \\ N, & for\ inner\ unit\ disk. \end{cases} \tag{13}$$

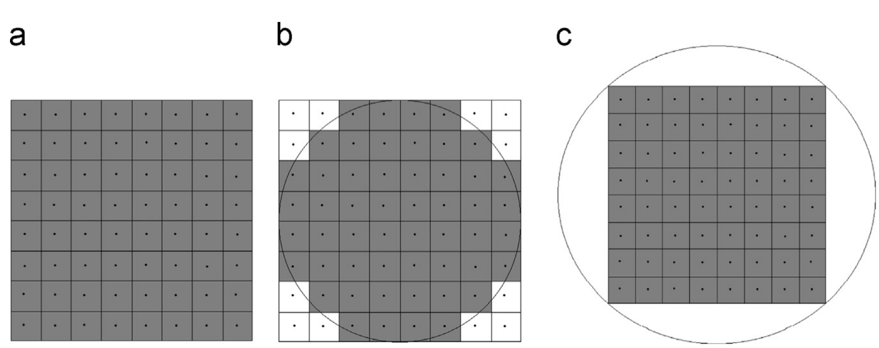

Fig. 1. (a) An 8×8 pixel grid, (b) inner unit disk mapping, and (c) outer unit disk mapping.