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# Fast computation of separable two-dimensional discrete invariant moments for image classification

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ABSTRACT

In this paper, we present a new set of bivariate discrete orthogonal polynomials based on the product of Meixner's discrete orthogonal polynomials by Tchebichef's, Krawtchouk's and Hahn's discrete orthogonal polynomials. This set of bivariate discrete orthogonal polynomials is used to define three new types of discrete orthogonal moments as Meixner–Tchebichef moments, Meixner–Krawtchouk moments and Meixner–Hahn moments. We also present an approach to accelerate the computation of these moments by using the image block representation for binary images and image slice representation for gray-scale images. A novel set of Meixner–Tchebichef invariant moments, Meixner–Krawtchouk invariant moments and Meixner–Hahn invariant moments is also derived. These invariant moments are derived algebraically from the geometric invariant moments and their computation is accelerated using an image representation scheme. The proposed algorithms are tested using several well-known computer vision datasets including, moment's invariability and pattern recognition. The performance of these invariant moments used as pattern features for a pattern classification is compared with the shape descriptors of Hu, Legendre, Tchebichef–Krawtchouk, Krawtchouk–Hahn and Tchebichef–Hahn invariant moments, the texture descriptors and the color descriptors for four different databases.


## 1. Introduction

From the early sixties, the theory of moments has been widely used for image analysis, classification and pattern recognition [1–5]. Hu was the first who used the geometric invariant moments in pattern recognition [1]. However, because of the non-orthogonal property of these geometrical moments, that causes the redundancy of information, Teague [3] in 1980 introduced the continuous orthogonal moments defined in terms of continuous orthogonal polynomials of Legendre and Zernike. The computation of continuous orthogonal moments requires discrete approximation of the continuous integral and discretization of the continuous space. This increases the computational complexity and causes the error of discretization [3–7]. To eliminate this error, scientists have introduced the discrete orthogonal moments such as Tchebichef [8], Krawtchouk [9] and Hahn [10] in field of image analysis. The use of discrete orthogonal moments based on the discrete orthogonal polynomials eliminates the need for numerical approximation and satisfies the orthogonal property [11]. All of the continuous and the discrete orthogonal moments have separable basic functions that can be expressed as two separate terms by producing the same two classical orthogonal polynomials with one variable. Recently, a novel set of discrete and continuous orthogonal moments based on the bivariate orthogonal polynomials have been introduced into the field of image analysis and pattern recognition [12]. These series of polynomials are solutions of second-order partial differential equations [13,14]. Koornwinder in [15] gave a general method of generating bivariate continuous orthogonal polynomials from continuous orthogonal polynomials with one variable. Dunkl and Xu in [16] have published an excellent paper of bivariate discrete orthogonal polynomials as a product of two families of classical discrete orthogonal polynomials with one variable. Zhu has studied in [12] seven types of continuous and discrete orthogonal moments based on the tensor product of two different orthogonal polynomials in one variable. The computation of orthogonal moments is limited by two major difficulties. The first is related to the high computational cost especially for large size images. The second is related to the propagation of numerical error in the computation of polynomials values [17]. To limit this error, Scientists apply the recurrence relation with respect to variable $x$ instead of order $n$ in the computation of discrete orthogonal polynomials [11–17]. To reduce the computational time cost of moments, several

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algorithms are introduced in literature [18–26]. If the focus was mainly given to the discrete and continuous moments based on the product of the same two classical orthogonal polynomials with one variable, no attention has been paid to accelerate the time computation of discrete orthogonal moments based on the product of two different discrete orthogonal polynomials with one variable. In this paper, we present a new set of discrete orthogonal moments based on the product of Meixner discrete orthogonal polynomials by Tchebichef, Krawtchouk and Hahn's discrete orthogonal polynomials, which are denoted Meixner–Tchebichef moments (MTM), Meixner–Krawtchouk moments (MKM), and Meixner–Hahn moments (MHM) respectively. We also present an approach to accelerate the time computation of MTM, MKM and MHM discrete orthogonal moments. Indeed, we propose a fast method for computing the bivariate discreet orthogonal polynomials of Meixner–Tchebichef, Meixner–Krawtchouk and Meixner–Hahn based on the use of recurrence relation with respect to variable $x$ instead of order $n$. Furthermore, we present a new method to compute MTM, MKM and MHM discrete orthogonal moments by describing an image with a set of blocks instead of individual pixels. Two algorithms of image block representation IBR [18] for binary images and image slice representation ISR [22] for gray-scale images are proposed. The paper also tests the ability of proposed discrete orthogonal moments in pattern recognition and object classification. For the purpose of object classification, it is vital that MTM, MKM and MHM discrete orthogonal moments must be independent of rotation, scaling and translation of the image. For this, we have derived a new set of discrete invariant moments of Meixner–Tchebichef (MTMI), Meixner–Krawtchouk (MKMI) and Meixner–Hahn (MHMI) under translation, scaling and rotation of the image. The discrete invariant moments MTMI, MKMI and MHMI are derived algebraically from the geometric invariant moments. A fast computation algorithm of this set of MTMI, MKMI and MHMI discrete orthogonal invariant moments is also presented using the image block representation method. The accuracy of object classification by our proposed MTMI, MKMI and MHMI discrete invariant moments is compared with the shape descriptors of Hu [1], Legendre [27], Tchebichef–Krawtchouk, Krawtchouk–Hahn and Tchebichef–Hahn [12] invariant moments, the color descriptors [36,37] and the texture descriptors [38,39].

The rest of the paper is organized as follows: In Section 2, we present the known results of the classical discrete orthogonal polynomials with one variable. These previous studies serve as a basic background for the rest of this paper. Section 3 is devoted to introduce discrete orthogonal polynomials in two variables. Section 4 presents a fast method to calculate MTM, MKM and MHM discrete orthogonal moments for binary and gray-scale images. Section 5 focuses on the deriving MTMI, MKMI and MHMI invariant moments from the geometric moments by two methods. Section 6 provides some experimental results concerning the classification of objects and the reduction of the time calculation of MTMI, MKMI and MHMI discrete orthogonal invariant moments. Section 7 concludes the work.

## 2. Classical discrete orthogonal polynomials of one variable

In this section, we will present a brief introduction of the theoretical background of discrete orthogonal polynomials with one variable of Tchebichef, Krawtchouk, Hahn and Meixner [25,26].

### 2.1. Krawtchouk polynomials

The discrete orthogonal polynomials of Krawtchouk with one variable $K_n(x;p,N)$ satisfy the following first-order partial difference equation [25].

$$(1-p)x\Delta\nabla K_n(x;p,N)+(Np-x)\Delta K_n(x;p,N)+nK_n(x;p,N)=0 \tag{1}$$

with

$$x,n=0,1,2\ldots N-1 \ \ and \ \ 0\prec p\prec 1$$

The $n$th Krawtchouk polynomials is defined by using hypergeometric function as

$$K_n(x;p,N)={}_2F_1(-n,x;-N;1/p)=\sum_{i=0}^{n}\varepsilon_{m,j}^{(p)}x^i \tag{2}$$

where ${}_2F_1$ is the generalized hypergeometric function given by

$${}_2F_1(a_1,a_2;b_1;z)=\sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k}{(b_1)_k}\frac{z^k}{k!} \tag{3}$$

The normalized discrete orthogonal polynomials of Krawtchouk are defined as

$$\tilde{K}_n(x;p,N)=K_n(x;p,N)\sqrt{\frac{w_k(x)}{\rho_k(n,N)}} \tag{4}$$

with $w_k(x)$ is the weight function of Krawtchouk discrete orthogonal polynomials

$$w_k(x;p,N)=\binom{N}{x}p^x(1-p)^{N-x} \tag{5}$$

and $\rho_k(n)$ is the squared norm of Krawtchouk discrete orthogonal polynomials defined as

$$\rho_k(n,N)=(-1)^n\left(\frac{1-p}{p}\right)^n\frac{n!}{(-N)_n} \tag{6}$$

The Pochhammer symbol is defined by $(x)_0=1$ and $(x)_k=x(x+1)\ldots(x+k-1)$ ; $k\geq 1$. The normalized discrete orthogonal polynomials of Krawtchouk $\tilde{K}_n(x;p,N)$ satisfy an orthogonal relation of the form

$$\sum_{x=0}^{N}\tilde{K}_n(x;p,N)\tilde{K}_m(x;p,N)=\delta_{nm} \tag{7}$$

$\delta_{nm}$ denotes the Dirac function.

To calculate the values of Krawtchouk's discrete orthogonal polynomials, we will use the recurrence relations of three-terms with respect to the order $n$. Indeed, Krawtchouk's discrete polynomials satisfy the following three-term recurrence relations with respect to order $n$ [25]:

$$\tilde{K}_n(x;p,N)=\frac{BD}{A}\tilde{K}_{n-1}(x;p,N)-\frac{CE}{A}\tilde{K}_{n-2}(x;p,N) \tag{8}$$

The initial values of Krawtchouk's polynomials are defined as follows:

$$\tilde{K}_0(x;p,N)=\sqrt{\frac{w_k(x;p,N)}{\rho_k(0,N)}}$$
$$\tilde{K}_1(x;p,N)=(-p(N-x)+x(1-p))\sqrt{\frac{w_k(x;p,N)}{\rho_k(1,N)}} \tag{9}$$

The coefficients $A$, $B$, $C$, $D$ and $E$ are given as follows:

$$A=n;\ \ B=x-n+1-p(N-2n+2);\ \ C=-p(1-p)(N-n+2)$$

$$D=\sqrt{\frac{n}{p(1-p)(N-n+1)}};\ \ \text{and}\ \ E=\sqrt{\frac{n(n-1)}{(p(1-p))^2(N-n+2)(N-n+1)}} \tag{10}$$

The calculated values of Krawtchouk's discrete orthogonal polynomials using the hypergeometric functions and recurrence relations of three-terms with respect to the order $n$ are complex and require a high computation time, especially for higher orders, which cause the propagation of numerical errors [17]. To limit this error, we applied the recurrence relation with respect to variable $x$