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Edges, transitions and criticality

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ABSTRACT

In this article, various notions of edges encountered in digital image processing are reviewed in terms of compact representation (or completion). We show that critical exponents defined in Statistical Physics lead to a much more coherent definition of edges, consistent across the scales in acquisitions of natural phenomena, such as high resolution natural images or turbulent acquisitions. Edges belong to the multiscale hierarchy of an underlying dynamics, they are understood from a statistical perspective well adapted to fit the case of natural images. Numerical computation methods for the evaluation of critical exponents in the non-ergodic case are recalled, which apply for the vast majority of natural images. We study the framework of reconstructible systems in a microcanonical formulation, show how it redefines edge completion, and how it can be used to evaluate and assess quantitatively the adequation of edges as candidates for compact representations. We study with particular attention the case of turbulent data, in which edges in the classical sense are particularly challenged. Tests are conducted and evaluated on a standard database for natural images. We test the newly introduced compact representation as an ideal candidate for evaluating turbulent cascading properties of complex images, and we show better reconstruction performance than the classical tested methods.

1. Introduction

As algorithms dedicated to the computation of edges in digital images emerged [1–8], Torre and Poggio [9], while observing that most methods rely on the ill-posed problem of differentiating digital images, proposed a general qualitative description of *edges*: they noted that edges are naturally associated to the concepts of *compact representation* (they call it *completion*), i.e., edges encode most information of an image [36]. Similarly, other authors note that edges represent an image's independent feature [12]. In [9] the authors focus on edge detection as the process of computing derivatives, and, while attempting to do so in a well-posed form, they are led naturally to the problem of prefiltering the image by a (e.g., Gaussian) kernel, which transforms the input signal into a differentiable mapping in the continuous domain, hence allowing the characterization of edges by differential operators. An instance of this formalism is the zero-crossing of second-order derivatives, as in [5–7,31,33,35], to cite a few, including a recent nonlinear derivative approach (called NLFS) [32]. This formal setting allowed the development of edge characteristics in the framework of differential geometry, a perspective that has become pervasive in image processing [10,23]. The multiscale nature of edges was recognized very early and it was noted that tracing edge properties across scales would gain insight into the physical process behind image formation. Neurophysics was demonstrating that, in the optical pathway, spatial filters of different sizes operate at the same location [11]. This is related to the processing of information in the early visual system [15], where cells tend to take advantage of the statistical regularities of the input signal in order to get compact representations out of redundancy [16,17].

The convolution of the input image signal by a Gaussian kernel introduces a scale parameter (the standard deviation of the Gaussian kernel) corresponding to a simple linear scale-space associated to the heat equation. This is often used as an argument for advocating multiscale properties of Gaussian prefiltering [9,22,25]. In general, however, the multiscale properties of complex systems do not comply with such an extreme simplification [18]. The advent of scale-space theory in Computer Vision allowed more complex multiscale representations corresponding, among others, to anisotropic diffusion schemes [19,24,34], which can incorporate probabilistic models of both sensor noise and operators' responses (to better estimate the gradient's magnitude threshold in case of noise). However, the simple example of an image corresponding to the acquisition of a turbulent fluid, like, for instance, a remotely sensed acquisition over the oceans, contains coherent structures associated to the cascading properties of intensive variables in Fully Developed Turbulence (FDT)

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[29]. It has an associated multiscale hierarchy consisting of sets having a multifractal nature [21] and, as such, cannot be contemplated within a differentiable scale-space framework. Incidentally note that in [30] authors write that an appropriate spatial scale depends upon the local structure of the edge, and thus varies *unpredictably* over the image.

In a seminal paper, Mallat and Zhong [26] relate multiscale Canny edge detection to the local maxima of a wavelet transform and study the completion of multiscale edges associated to the maxima of wavelet coefficients (multiscale edge detection [26,37]); for that purpose they introduce a reconstruction algorithm of a signal from its edges. The quality of the reconstruction is quantitatively evaluated by the SNR of the original and reconstructed signals, hence providing an accurate evaluation of the quality of *edge pixels* determined by their method; this evaluation is different from the previous criteria used in computing edge detection performance [10]. Local maxima of wavelet coefficients are also used by other authors to form the basis of the Wavelet Transform Modulus Maxima (WTMM) methodology [27]. Edges can also be understood as alignment of Fourier or wavelet phases across scales [13,14].

In this paper, we show that recent developments around the notion of *transition* in nonlinear physics, along with enhanced computational methods of its quantitative parameters (most notably *singularity exponents*) [29], lead to a notion of edge that provides better results over all the previous declined versions encountered in image processing w.r.t. edge completion. Our results strongly advocate for a definition of edge based on nonlinear operators while we prove along the way, and incidentally, that previous nonlinear approach [32] also works better than the classical ones w.r.t. to reconstruction of an image from its edge data. This can be done by referring to the early-addressed pertinent notion of *compact representation* (completion). When neurophysics and the study of biological vision in mammals state that edges encode most information in an image signal, this must have the consequence of being able to reconstruct accurately an image from the compact representation of its edge pixels ([37], p. 194). Statistically, image acquisitions correspond to processes out of the equilibrium state, so that, from a theoretical point of view, transitions associated to scale-space formulations reviewed above cannot be correct. We show that the new notion of edge outperforms the most well-known previous ones and that it is naturally robust to noise. We give specific attention to the case of turbulent images, whose edges are not well defined in the classical context of edge detection, and we show that in this context the new notions introduced in this article work much better than the previous ones.

Unpredictability of edges, the concept of singularity exponents and the framework of reconstructible systems are introduced in Section 2. Edge consistency across the scales is addressed in Section 3. Quantitative results are shown and discussed in Sections 4 and 5, respectively, where in Section 4.4 the case of turbulent images is specifically addressed. In Section 6 conclusion follows.

2. Edges, unpredictability and reconstructability revisited using the microcanonical multiscale formalism (MMF)

In this section we show how the microcanonical multiscale formalism (MMF [47]) can be applied to edge detection and image reconstruction. We will show that ideas borrowed from Statistical Physics about criticality and exponents, when evaluated in a microcanonical formulation, are associated to a computable notion of *transition*, intimately related to *predictability* in complex systems [38]. This in turn gives rise to a notion of edge whose quantitative performance evaluation can be tested through the framework of *reconstructible systems*.

2.1. Local predictability exponents

Classically, edges are related to sharp variations of the image gradient. The main idea worked out in this article is to delve deeper into developing the notion of "sharp variation", and relate it to the more general notion of "transition" defined in Statistical Physics for *intensive* physical variables. For that matter, a scalar image I defined over a compact subset of $\mathbb{R}^2$ is identified with an intensive physical variable (such an identification corresponds exactly to the physics of the acquisition for images of natural phenomena such as in infrared remote sensing imagery). In nonlinear physics, the relation between the transitions of an intensive variable and criticality is well known, explained, and quantitatively formalized through the notion of *critical exponent* [39]. We recall that definition here. We say that image I has a critical exponent $h(\vec{x})$ at point $\vec{x}$, if for at least one multiscale functional $\mathbb{T}_r$,[1] dependent on scale r, the following equation holds:

$$\mathbb{T}_r I(\vec{x}) = \alpha(\vec{x}) r^{h(\vec{x})} + o(r^{h(\vec{x})}) \; (\vec{r} \to 0) \tag{1}$$

where the term $o(r^{h(\vec{x})})$ is a quantity that decreases to zero faster than $r^{h(\vec{x})}$ when r goes to 0 and $\alpha(\vec{x})$ is a signal-dependent amplitude prefactor. An effective choice for the functional $\mathbb{T}_r$ leads to a measure given by the total variation of the image gradient [47], and is defined as follows:

$$\mu(\mathcal{B}_r(\vec{x})) = \int_{\mathcal{B}_r(\vec{x})} \| \nabla I \| (\vec{x}) \, \mathrm{d}(\vec{x}) \tag{2}$$

where $\mathcal{B}_r(\vec{x})$ is a ball of radius r centred at point $(\vec{x})$ of the signal domain. The wavelet transform of the measure μ then allows us to infer a more computable version of the singularity exponents (less prone to noise variation) [48], such that

$$\mathcal{T}_{\Psi} \mu(\vec{x}) = \alpha_{\Psi}(\vec{x}) r^{h(\vec{x})} + o(r^{h(\vec{x})}) (r \to 0) \tag{3}$$

where Ψ is an associated wavelet.

The central problem is to compute at high numerical precision the value of $h(\vec{x})$ at point $\vec{x}$: bad approximations of singularity exponents lead to poor reconstructions. We will address the problem of robust computation of singularity exponents in Section 2.2. For the moment we note that Eq. (1) is a pointwise and localized version of the definition used in introducing singularity spectrum [27,46]: we do not make use of statistical averages and grand ensembles, but seek to evaluate $h(\vec{x})$ at point $\vec{x}$ (which means that we cannot rely on stationarity hypothesis). We denote $\mathcal{F}_h$ the component in the image's domain associated to singularity exponent value h as follows:

$$\mathcal{F}_h = \{\vec{x} : h(\vec{x}) = h\} \tag{4}$$

This family of sets is naturally associated to the multiscale hierarchy in an image [47]. In the case of natural images of the physical world, it is expected that the values of $h(\vec{x})$ are bounded by below, so that the *Most Singular Manifold* or MSM can be defined as:

$$\mathcal{F}_\infty = \{\vec{x} : h(\vec{x}) = h_\infty = \mathrm{Min}(h(\vec{x}))\} \tag{5}$$

noting that, in digital signals, the value h_∞ is thresholded and must correspond to a (small) tolerance interval. The MSM comprises the set of points in an image with the sharpest transitions,

[1] Typical examples of $\mathbb{T}_r$ can be wavelet transforms at scale r or some differential operators applied to the signal.