Pattern Recognition 48 (2015) 2227–2240

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Similarity transformation parameters recovery based on Radon transform. Application in image registration and object recognition

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ARTICLE INFO

Article history:
Received 20 February 2014
Received in revised form
22 June 2014
Accepted 23 January 2015
Available online 2 February 2015

Keywords:
Radon transform
2π-Based Radon transform
Rotation
Scaling and translation transforms
Parameters recovery algorithm
Additive image noise

ABSTRACT

The Radon transform, since its introduction in the beginning of the last century, has been studied deeply and applied by researchers in a great number of applications, especially in the biomedical imaging fields. By using the Radon transform properties, the issue is to recover the transformation parameters regarding the rotation, scaling and translation, by handling only the image projections assuming no access to the spatial domain of the image. This paper proposes an algorithm using an extended version of the Radon transform to recover such parameters relating to two unknown images, directly from their projection data. Especially, our approach deals with the problem of the estimation accuracy of the rotation angle and its finding in one step instead of two steps as it is reported in the literature. This method may be applied in image registration as well in object recognition. The results are, for the first time, exploited in object recognition where comparison with powerful descriptors shows the outstanding performance of the proposed paradigm. Moreover, the influence of additive noise on registration and recognition experiments is discussed and shows the efficiency of the method to reduce the effect of the noise.

1. Introduction

The estimation of the affine transformation is very useful for the building of invariant vision systems, on which are based many computer vision problematics, such as, image registration and object recognition. These disciplines play an important role in computer vision applications (e.g. medical imaging, automatic target recognition, and industrial inspection) where the determination of the spatial transformations between two images is often the keystone of some existing methods. This spatial transformation has as role, for image registration, to bring homologous points in registered images into correspondence [1], whilst for object recognition, it permits to match two objects. Among other spatial transformations, the similarity transforms, i.e. rotation, scaling and translation (RST) are often involved in many applications, where the object shape should be preserved, such as (1) analysis of pictures captured by camera in a common reference frame, (2) state change quantification of a moving target and (3) weld defect identification [2] on radiograms in nondestructive testing (NDT).

Although on many decades, methods and techniques such as invariant moments and invariant correlation have carried out noticeable progress for RST parameters estimation, it is only very recently that the methods based on the Radon transform [3] have received gain of interest. In fact, working directly with the data provided by the Radon projections allows us to avoid image reconstruction techniques like filtered back projection, which are computationally expensive and prone to reconstruction artifacts [4]. Computed tomography (CT) is an essential imaging technology in medicine, NDT and materials research [5], examples of application where the determination of the internal structure of an object is closely associated with the Radon transform; and only the projection data (often represented as sinograms) are available to the imaging system manipulator. The techniques designed to recover spatial transformations linking two patterns or two images using directly their Radon transforms have received recently a lot of interest and are increasingly investigated. For image registration tasks [6], the authors in [7] estimate the RST parameters between a reference image and a distorted image. Here, only the rotation angle is estimated using Radon transform. Knowing that the sinograms of both images are the same, except for the circular shift along the direction of the angular component (ϕ), they use the cross-correlation on the rows of the Radon matrix to recover the rotation angle. However, results remain fairly weak on some images. For the tomographic registration

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http://dx.doi.org/10.1016/j.patcog.2015.01.017
0031-3203/

purpose, the algorithm in [8] uses some properties of the Radon and Fourier transforms, such as the central slice and Fourier shift theorems to identify translational and angular offsets and then, the fitting is obtained by using the cross-correlation method. Related to the NDT community, the work in [9] is devoted to the application of the Fourier phase matching method in the registration of radiography and computed tomography projections which has the potential to allow automated metrology and defect detection. Still using image projections, the authors in [5] develop a method which incorporates all degrees of freedom of an affine transformation. The parameters of the latter are estimated as an optimization problem where a trust region-based optimizer is used [10]. However, the mean relative magnitude error rely high around 10% and 5° for the mean angular error, for both cases of two-dimensional parallel and fan beam geometries. In [11], the authors present a method which consists to estimate transformation parameters for a binary object subject to reflection, scaling, translation and rotation using only the Radon projections. Nevertheless, the objective function, as it is formulated by the authors, cannot be generalized for any rotation angle in the range $[0, 2\pi]$. In [12], the authors use the results obtained in [11] to identify the transform parameters for a fast matching algorithm. However, the rotation angle estimation is not very accurate compared to [11] and needs two running steps to distinguish the rotation angle ϕ_0 from $\phi_0+\pi$.

In this paper, we develop a method in order to estimate the RST transforms relating two unknown images, directly from their projection data. A preliminary study of the work reported in this paper has already been published in [13]. Here, we propose a deep analysis on theoretical and experimental aspects. As first step, we define the 2π-based Radon transform to deal with the problem of rotation by any angle belonging in $[0, 2\pi]$, unlike [11]. The resulting algorithm may be used on sinograms obtained from spatial images as well on projections considered as raw data. Consequently, this method may be introduced in an intensity-based image registration working in the space domain and where the mapping model consists in a similarity transform. Moreover, this method is originally applied in object recognition where the objective function used for the rotation angle estimation is taken as a similarity measure. In addition to recognize object through a pattern classification task, the main advantage of the proposed method is that the RST transform parameters between two matched patterns are implicitly recovered without any additional computing. However, the major part of methods using Radon transform devoted to invariant pattern recognition, e.g. [14–16], bring together similar patterns in the same class but are not able to provide the estimation of the geometric transforms giving the best similarity between two patterns.

The remainder of the paper is organized as follows. Section 2 gives the basic material to understand the main properties of the Radon transform. Section 3 is devoted to the problem statement whereas in Section 4, the 2π-based Radon transform is defined and its utilization is motivated. In Section 5, the proposed RST parameters recovery method is detailed and summarized. The experimental results are discussed in Section 6. Finally, the conclusion and some future directions of this work are drawn in Section 7.

2. Basic material

To define the Radon transform (RT) [3], let L be a straight line in the x–y plane and ds be an increment of length along L. Then, the Radon transform of a real valued function f, denoted $f^\vee$, is defined by its integral as

$$f^\vee(p,\phi)=\int_L f(x,y)\,\mathrm{d}s \tag{1}$$

$f^\vee(p,\phi)$ is determined by integration of all lines $L_{p,\phi}$, in the x–y

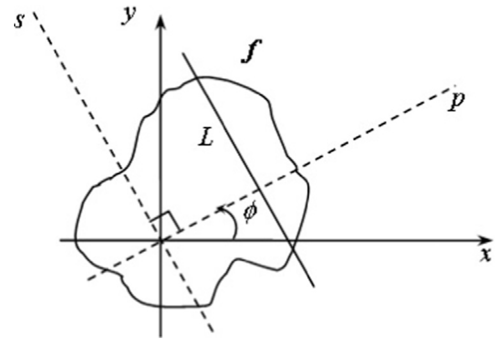

Fig. 1. Definition of the Radon transform.

plane, $p\in\mathbb{R}$, $\phi\in[0,\pi]$. From Fig. 1, the equation of L is given by $p=x\cos\phi+y\sin\phi$. If we rotate the axes by an angle ϕ, and label the new axes p and s, we obtain

$$x=p\cos\phi-s\sin\phi,\quad y=p\sin\phi+s\cos\phi \tag{2}$$

Then, $f^\vee$ is defined by

$$f^\vee(p,\phi)=\int_{-\infty}^{\infty} f(p\cos\phi-s\sin\phi,\,p\sin\phi+s\cos\phi)\,\mathrm{d}s \tag{3}$$

The most useful properties of RT in this work are

- *Linearity*: $(f+h)^\vee(p,\phi)=f^\vee(p,\phi)+h^\vee(p,\phi)$. RT is linear.
- *Periodicity*: $f^\vee(p,\phi)=f^\vee(p,\phi\pm 2k\pi)$, $k\in\mathbb{Z}$. RT is 2π-periodic
- *Symmetry*: $f^\vee(p,\phi)=f^\vee(-p,\phi\pm(2k+1)\pi)$, $k\in\mathbb{Z}$. RT is semi-symmetric.
- *Translation by a vector* $\vec{u}=(x_0,y_0)$: $h=T_{(x_0,y_0)}(f)\Rightarrow h^\vee(p,\phi)=f^\vee(p-x_0\cos\phi-y_0\sin\phi,\phi)$.
- *Rotation by an angle* ϕ_0: $h=R_{\phi_0}(f)\Rightarrow h^\vee(p,\phi)=f^\vee(p,\phi-\phi_0)$.
- *Scaling by a factor* α $(\alpha>0)$: $h=S_\alpha(f)\Rightarrow h^\vee(p,\phi)=\alpha f^\vee(p/\alpha,\phi)$.

3. Problem statement

We consider the case of 2D mono-channel images. Assume f as an image in x–y plane, subjected to a sequence of affine transformations R_{ϕ_0}, S_α, and T_{x_0,y_0}, where ϕ_0 (in $[0,2\pi]$), α (in $\mathbb{R}_+$) and (x_0,y_0) (in $\mathbb{R}^2$) are, respectively, the rotation angle, the scaling factor and the translation vector components. The transform image, noted g with coordinates x', y' is a composition of three functions formulated as

$$g=T_{x_0,y_0}\circ S_\alpha\circ R_{\phi_0}[f] \tag{4}$$

or as a matrix product using homogeneous coordinates

$$\begin{bmatrix}x'\\y'\\1\end{bmatrix}=\begin{bmatrix}1&0&x_0\\0&1&y_0\\0&0&1\end{bmatrix}\begin{bmatrix}\alpha&0&0\\0&\alpha&0\\0&0&1\end{bmatrix}\begin{bmatrix}\cos\phi_0&-\sin\phi_0&0\\\sin\phi_0&\cos\phi_0&0\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix} \tag{5}$$

which leads to the following equation system:

$$x'=\alpha(\cos\phi_0 x-\sin\phi_0 y)+x_0,\quad y'=\alpha(\sin\phi_0 x+\cos\phi_0 y)+y_0 \tag{6}$$

Let $c_f(x_f,y_f)$ and $c_g(x_g,y_g)$ be, respectively, the centroids of f and g, then the total translation $\overline{c_f c_g}$ which results from the algebraic sum of the translation itself and the translation due to rotation and scaling has as coordinates

$$\begin{cases}x_{0t}=x_g-x_f=(\alpha\cos\phi_0-1)x_f-\alpha\sin\phi_0\;y_f+x_0\\ y_{0t}=y_g-y_f=\alpha\sin\phi_0\;x_f+(\alpha\cos\phi_0-1)y_f+y_0\end{cases} \tag{7}$$