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Variational image segmentation model coupled with image restoration achievements

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ABSTRACT

Image segmentation and image restoration are two important topics in image processing with a number of important applications. In this paper, we propose a new multiphase segmentation model by combining image restoration and image segmentation models. Utilizing aspects of image restoration, the proposed segmentation model can effectively and robustly tackle images with a high level of noise or blurriness, missing pixels or vector values. In particular, one of the most important segmentation models, the piecewise constant Mumford–Shah model, can be extended easily in this way to segment gray and vector-valued images corrupted, for example, by noise, blur or information loss after coupling a new data fidelity term which borrowed from the field of image restoration. It can be solved efficiently using the alternating minimization algorithm, and we prove the convergence of this algorithm with three variables under mild conditions. Experiments on many synthetic and real-world images demonstrate that our method gives better segmentation results in terms of quality and quantity in comparison to other state-of-the-art segmentation models, especially for blurry images and those with information loss.

1. Introduction

Image segmentation and image restoration are two important concepts in image processing. Image segmentation consists in partitioning a given image into multiple segments to transfer the representation of the image into a more meaningful one which is easier to analyze. It is typically used to locate objects and boundaries within an image. Image restoration is the operation of estimating a desired clean image from its corrupted version. Corruption may come in many forms, such as blur, noise, camera misfocus, or information loss. Obviously, image segmentation can be used as preprocessing or postprocessing of image restoration. In other words, these two topics influence each other substantially.

Let $\Omega \subset \mathbb{R}^2$ be a bounded, open, connected set, and $f : \Omega \to \mathbb{R}$ a given image. Without loss of generality, we restrict the range of f to [0,1]. Let $g : \Omega \to \mathbb{R}$ denote the desired clean image, then $f = g + n_f$, where n_f is the additive noise. Many image restoration models can be written in the form

$$E(g) = \mu \Phi(f,g) + \phi(g), \tag{1}$$

where $\Phi(f,g)$ is the data fidelity term, $\phi(g)$ is the regularization term, and $\mu > 0$ is a regularization parameter balancing the trade-off between terms $\Phi(f,g)$ and $\phi(g)$. If set $\Phi(f,g) = \int_\Omega (f-g)^2\, dx$ and $\phi(g) = \int_\Omega |\nabla g|\, dx$ (total variation term), then model (1) becomes the ROF model proposed by Rudin, Osher and Fatemi in 1992 [37], i.e.,

$$E(g) = \mu \int_\Omega (f-g)^2\, dx + \int_\Omega |\nabla g|\, dx. \tag{2}$$

One important advantage of model (2) is that it preserves the edge information of f very well, but it also introduces the staircase effect. Many previous attempts to remove the staircase effect are based on higher-order derivative terms, see [5,14,29,38,45]. For example in [5], tight-frame technic was used in $\phi(g)$ to obtain more details of higher-order derivative information of f. The relationship between total variation and tight-frame can be found in [41]. As we know, the data fidelity term $\Phi(f,g) = \int_\Omega (f-g)^2\, dx$ is especially effective for Gaussian noise [37]. For removing Poisson noise, $\Phi(f,g) = \int_\Omega (g - f \log g)\, dx$ is proposed [17], whilst $\Phi(f,g) = \int_\Omega |f-g|\, dx$ is proposed for removing impulsive noise [31]. Refer to [1,11,12,17,25,31,27,39,40] and references therein for the details of Poisson and impulsive noise removal. Note that the image restoration model (1) can be extended to process blurry images after introducing a problem related linear operator $\mathcal{A}$ in front of g [36].

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Let $\Gamma \in \Omega$ represent the boundary of the objects within an image, and Ω_i be the parts of the segmented objects fulfilling $\Omega = \bigcup_i \Omega_i \cup \Gamma$. The Mumford–Shah model is one of the most important image segmentation models, and has been studied extensively in the last twenty years. More precisely, in [30], Mumford and Shah proposed an energy minimization problem which approximates the true solution by finding optimal piecewise smooth approximations. The energy minimization problem was formulated as

$$E(g,\Gamma) = \frac{\lambda}{2}\int_{\Omega}(f-g)^2\,dx + \frac{\mu}{2}\int_{\Omega\setminus\Gamma}|\nabla g|^2\,dx + \text{Length}(\Gamma), \tag{3}$$

where λ and μ are positive parameters, and $g:\Omega\to\mathbb{R}$ is differentiable in $\Omega\setminus\Gamma$ but may be discontinuous across Γ. Because model (3) is nonconvex, it is very challenging to find or approximate its minimizer, see [8,9,22]. Many works [26,44] concentrate on simplifying model (3) by restricting g to be a piecewise constant function ($g = c_i$ in Ω_i), i.e.,

$$E(g,\Gamma) = \frac{\lambda}{2}\sum_{i=1}^{K}\int_{\Omega_i}(f-c_i)^2\,dx + \text{Length}(\Gamma), \tag{4}$$

where K is the known number of phases of f. Using the coarea formula [20], and recognizing that the total variation of the characteristic function of a set is its perimeter [20,21], model (4) can be rewritten as

$$E(c_i,u_i) = \lambda\sum_{i=1}^{K}\int_{\Omega}(f-c_i)^2 u_i\,dx + \sum_{i=1}^{K}\int_{\Omega}|\nabla u_i|\,dx,$$
$$\text{s.t.}\quad \sum_{i=1}^{K}u_i(x) = 1,\quad u_i(x)\in\{0,1\},\quad \forall x\in\Omega. \tag{5}$$

Moreover, model (5) with $K=2$ is the Chan–Vese model [16], and with fixed c_i is a special case of the Potts model [34]. Due to the nonconvex property of (5), the exact convex version of (5) was proposed when $K=2$ and c_i fixed [13]. For $K>2$, recent authors have focused on relaxing u_i and solving the following model:

$$E(u_i,c_i) = \lambda\sum_{i=1}^{K}\int_{\Omega}(f-c_i)^2 u_i\,dx + \sum_{i=1}^{K}\int_{\Omega}|\nabla u_i|\,dx,$$
$$\text{s.t.}\quad \sum_{i=1}^{K}u_i(x) = 1,\quad u_i(x)\ge 0,\quad \forall x\in\Omega. \tag{6}$$

Refer to [2,24,28,33,46] and references therein for more details. One drawback of model (6) is that *it is not good at segmenting images corrupted by blur or information loss*, and this is one of the main problems to solve in this current paper.

In [32], a model of coupling image restoration and segmentation based on a statistical framework of generalized linear models was proposed, but the analysis and algorithm therein are only focused on the two-phase segmentation problem. In our previous work [6], a two-stage segmentation method which provides a better understanding of the link between image segmentation and image restoration was proposed. The method suggests that for segmentation, it is reasonable and practicable to extract different phases in f by using image restoration methods first and thresholding second. Moreover, in our recent work [7], we proved that the solution of the Chan–Vese model [16] for certain λ can actually be given by thresholding the minimizer of the ROF model (2) using a proper threshold, which clearly provides one kind of relationship between image segmentation and image restoration.

In this paper, we propose starting with the extension of the piecewise constant Mumford–Shah model (4) to manage blurry image, and then applying a novel segmentation model by composing model (4) with a data fidelity term borrowed from the field of image restoration. Since we only add a new fidelity term, which usually possesses good properties such as differentiability, the solution of the proposed model is not more involved compared with solving model (4). It can be solved efficiently using the alternating minimization (AM) algorithm [18] with the ADMM or primal–dual algorithms [4,10,23]. We prove that under mild conditions, the AM algorithm converges for the proposed model. The proposed model can segment blurry images easily but model (4) cannot. Moreover, it can also deal with images with information loss and vector-valued images (e.g., color images). Due to the advantage of two data fidelity terms, one from image restoration and the other from image segmentation, our model is much more robust and stable. Experiments on many kinds of synthetic and real-world images demonstrate that our method gives better segmentation results in comparison with other state-of-the-art segmentation methods, especially in the case of blurry images and images with information loss.

Contributions: The main contributions of this paper are summarized as follows.

(1) Coupling variational image segmentation models and image restoration models, which provides a new methodology for multiphase image segmentation.
(2) Extending the piecewise constant Mumford–Shah model (4) by incorporating image restoration achievements so that the new constructed variation segmentation model can handle blurry images easily.
(3) Thanks to the image restoration achievements, the new variation segmentation model has the potential to process many different types of noise, for example Gaussian, Poisson and impulsive noise.
(4) Different kinds of vector-valued image, for example color images, and images with information loss, are also covered in the proposed variational segmentation model.
(5) The convergence of the AM algorithm with three variables to the proposed variational model is proved.

The rest of this paper is organized as follows. In Section 2, we propose our new segmentation model and extend it so that it can deal with vector-valued images and images with some pixel values missing. An AM algorithm for our model is introduced in Section 3. The convergence of it will be proved in Section 4. In Section 5, we compare the performance of our proposed method with state-of-the-art multiphase segmentation methods on various synthetic and real-world images. Conclusions are drawn in Section 6.

2. The proposed variational image segmentation model

We propose our image segmentation model by combining the piecewise constant Mumford–Shah model (5) with the fidelity term $\Phi(f,g)$, which comes from the image restoration model (1). More precisely, our proposed segmentation model aims to minimize the energy

$$E(u_i,c_i,g) = \mu\Phi(f,\mathcal{A}g) + \lambda\Psi(g,u_i,c_i) + \sum_{i=1}^{K}\int_{\Omega}|\nabla u_i|\,dx,$$
$$\text{s.t.}\quad \sum_{i=1}^{K}u_i(x) = 1,\quad u_i(x)\in\{0,1\},\quad \forall x\in\Omega, \tag{7}$$

where $g\in L^2(\Omega)$ and $\mathcal{A}$ is the problem-related linear operator. For example, $\mathcal{A}$ can be the identity operator for a noisy observed image f or a blurring operator if there are both noise and blur in f. The blurring operator $\mathcal{A}$, for example Gaussian bur or motion blur, can be estimated by using image deblurring methods (refer to [35,43,47] and references therein). Therefore, researchers generally assume that $\mathcal{A}$ is known in image segmentation or image restoration. The first term $\Phi(f,\mathcal{A}g)$ is a data fidelity term arising from the image restoration model (1). It controls g to within a short distance of the given corrupted image f; in other words, it