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Image deblurring with matrix regression and gradient evolution

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ABSTRACT

This paper presents a supervised learning algorithm for image deblurring. The task is addressed into the conceptual framework of matrix regression and gradient evolution. Specifically, given pairs of blurred image patches and their corresponding clear ones, an optimization framework of matrix regression is proposed to learn a matrix mapping. For an image to be deblurred, the learned matrix mapping will be employed to map each of its image patches directly to be a new one with more sharp details. The mapped result is then analyzed in terms of edge profiles, and the image is finally deblurred in way of gradient evolution. The algorithm is fast, and easy to be implemented. Comparative experiments on diverse natural images and the applications to interactive deblurring of real-world out-of-focus images illustrate the validity of our method.

1. Introduction

Image deblurring is a classical problem that has been extensively studied in the circles of image processing, computer graphics and computer vision. Although great progresses have been achieved in the past years [1–9], this task still remains far from being solved for real-world applications. As an inverse problem, the main challenge lies in that it is under-constrained since we need to restore the high frequency (sharp) details from the ruined image. In the absence of prior knowledge about the blurring mechanism, this brings intrinsic ambiguity of modeling the blur function and restoring the needed details.

The efforts in image deblurring have surged in recent decades, with the development of numerous approaches and proposals for real-world occasions. Most early methods rely on the deconvolution tricks [10–12], such as Richardson–Lucy algorithm [13,14], Wiener filtering, least-squares deconvolution [10], and so on. The main shortcoming of deconvolution approaches lies in that the deblurring quality largely depends on the kernel estimate. In addition, different tools of mathematical analysis are also employed to deal with this task, typically including wavelet [15,16], variational [17], and regularization [18–22]. Along the line of deconvolution, some approaches are formulated in terms of blind deconvolution, and example methods can be found in [23–29,12,30]. In blind deconvolution, it is not easy to estimate a proper kernel that is well suited to the occasion.

Another family of deblurring algorithms have been developed, explicitly or implicitly, with prior knowledge to help reduce the degrees of freedom of the problem. Typically, natural image statistics are used as prior knowledge to guide the deblurring [31–33,8]. Based on the fact that the statistics of derivative filters on images may be significantly changed after blurring, Levin modeled the expected ones as a function of the width of blur kernel [2]. Fergus et al. proposed to recover the patch images by finding the values with highest probability guided by a prior on the statistics [34], which states that natural images obey heavy-tailed distributions of image gradients. Along this line, approaches of modifying the gradient fields have been proposed with gradient adaptive enhancement [35], gradient penalty by a hyper-Laplacian distribution [36], gradient projection [37], and so on. In addition, information related to sparse representation [27,38,36], color statistics [39], and multi-images [3,40], has also been used to improve the image quality.

The task of deblurring has also been addressed in view of statistic inference or machine learning. A popular modeling tool is the Bayesian framework [41,42,34,8]. Typically, Fergus et al. employed a Bayesian approach to estimate the blur kernel implied by a distribution of probable images [34]. Bayesian framework has also been used to find the most likely estimate of the sharp image [8]. Except the Bayesian frameworks, Su et al. constructed a hybrid learning system, in which both unsupervised and supervised learning methods are employed to deblur images [43]. Later, based on a pair of blurred images, Liu et al. proposed a non-blind deconvolution approach [3]. More recently, Kenig et al. proposed a subspace learning based framework to model the space of point spread functions [44]. To this end, they employed principal component analysis to learn the space from examples at hand [44]. Then, a blind deconvolution algorithm

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is developed for new image to be deblurred. During each iteration of blind deconvolution, a prior term is added to attract the desired point spread function to the learned point spread function space. The application of the proposed algorithm is demonstrated on three-dimensional images acquired by a wide-field fluorescence microscope, indicating its ability to generate restorations with high quality.

In the literature, regression has been applied to image restoration. Typically, Hammond and Simoncelli developed a general framework for combination of two local denoising methods [45]. In their framework, a spatially varying decision function is employed to balance the two methods. Fitting the needed decision function to data is then treated as a regression problem. Kernel ridge regression is finally used to achieve this goal [45].

Additionally, regression is also applied to image deblurring. Takeda et al. derived a deblurring estimator [6] in terms of kernel regression. By using the Taylor series, they implicitly assumed that the regression function is a locally smooth function up to some order. The locally weighted kernel regression is then employed to achieve this goal. As a whole, the image to be resorted is ordered into a column-stacked vector, and the optimization problem is constructed by integrating together the regression representations of all the pixels with matrix operator [6]. Formally, this will generate a large-scale optimization problem for large image. In addition, although kernel regression is used, their method is unsupervised as no prediction function is learned from samples at hand for new images.

This paper presents a supervised learning algorithm for image deblurring. Our algorithm is developed on the conceptual framework of *matrix regression* and *gradient evolution*. To our best knowledge, this is the first time that the conception of matrix regression (MR) is presented for image deblurring. In this framework, we do not estimate the blur kernel, but learn a matrix mapping to transform image patches to be the desired ones. To this end, a supervised learning algorithm of MR is proposed to learn the matrix mapping from the given set including blurred image patches and their corresponding clear ones. The learned matrix mapping will be used to map its patches of the image to be deblurred. The mapped result is then evolved in a gradient field constructed to enhance the edges of the final image. Comparative experiments illustrate the efficiency and effectiveness of the proposed method. Its applications to the interactive deblurring of out-of-focus images also indicate the validity of our method.

Specifically, the advantages or details of our method can be highlighted as follows:

(1) Beyond blur identification and deconvolution used popularly in existing deblurring algorithms, our algorithm is addressed into the supervised learning framework, namely matrix regression framework. Instead, the learned matrix mapping is used to transform the blurred image patches.
(2) The MR algorithm is formulated as an optimization problem. The optimum can be obtained within a few iterations. In each iteration, only two groups of linear equations should be solved. The scale of the linear equations is very small since it equals to the size of the training patches. As a result, the optimization problem can be efficiently solved.
(3) On the whole image level, the computational complexity of transforming the blurred image with the learned matrix mapping scales linearly in the number of image patches. Moreover, the gradient evolution can be fulfilled via pixel-wise update. Thus, the computational complexity is also linear in the number of pixels. Low computational complexity and low memory requirement will facilitate its real-world applications of our method.

The remainder of this paper is organized as follows. In Section 2, the MR framework is developed. Section 3 presents the gradient evolution and describes the deblurring algorithm. Section 4 reports the experimental results. Section 5 demonstrates the applications to interactive deblurring of real-world out-of-focus images. Conclusions will be drawn in Section 6.

2. Matrix regression

2.1. Problem formulation

Generally, the image blurring model can be formulated as follows:

$$\mathcal{G} = f * \mathcal{I} + n, \tag{1}$$

where $\mathcal{I}$ is the imaged objects, f denotes the imaging system, $\mathcal{G}$ is the acquired image, n stands for the pixelwise additive noise, and "$*$" is a convolution operator. In real world situations, blur often comes from two types: out-of-focus lens or motion. For example, f is usually assumed to be a Gaussian point spread function for deblurring out-of-focus images. Here our task is to restore $\mathcal{I}$ from $\mathcal{G}$.

As an inverse problem, the task is under-constrained as f is unknown. This includes two cases. One is that the type of the kernel is known, while its size and element values are unknown. Another is that the type, size and element values are all unknown. Actually, there are many possible solutions to problem (1). Thus, employing prior knowledge about the blur kernel is fundamentally necessary to help constrain the solution to the desired images.

Algorithmically, most existing approaches solve problem (1) with tricks of deconvolution or blind deconvolution, in which a blur kernel is estimated. Differently, we address this task into the framework of supervised learning in terms of matrix regression.

As a supervised learning problem, now the task can be formulated as follows. Suppose we are given N blurred image patches in $\mathcal{X} = \{\mathbf{A}_i\}_{i=1}^N \subset \mathbb{R}^{m \times n}$ and their corresponding clear patches in $\mathcal{Y} = \{\mathbf{B}_i\}_{i=1}^N \subset \mathbb{R}^{m \times n}$, our goal is to find a matrix mapping

$$\mathbf{B} = \mathbf{LAR}, \tag{2}$$

such that for each patch we have

$$\mathbf{B}_i = \mathbf{L}\mathbf{A}_i\mathbf{R} + \mathbf{n}, \quad i = 1, 2, \ldots, N. \tag{3}$$

where $\mathbf{A}$ and $\mathbf{B}$ are two $m \times n$ matrices, $\mathbf{n} \in \mathbb{R}^{m \times n}$ is a difference term (related to model errors or noises), $\mathbf{L}$ is an $m \times m$ matrix and $\mathbf{R}$ is an $n \times n$ matrix.

The motivation behind the use of the mapping in (2) can be explained as follows. Intrinsically, as a bilinear mapping, (2) can be viewed as a combination of row deblurring and column deblurring. As a whole, a linear restoration of patch $\mathbf{A}$ is achieved since $\mathbf{B}$ is a linear function of all of the elements in $\mathbf{A}$. In addition, beyond converting patches $\mathbf{A}$ and $\mathbf{B}$ into two column-stacked vectors $\mathbf{a} \in \mathbb{R}^{mn}$ and $\mathbf{b} \in \mathbb{R}^{mn}$ respectively, directly employing matrix mapping can facilitate the computation. For example, suppose the size of the blur kernel (2D filter) is 41×41. Then, totally there are 1681 coefficients in $\mathbf{b} = \mathbf{w}^T\mathbf{a}$ to be solved. To accurately estimate $\mathbf{w}$, one needs to prepare at least 1681 pairs of samples in $\mathcal{X}$ and $\mathcal{Y}$ (that is, $N \geq 1681$). Otherwise, we will obtain an under-determined problem. In contrast, with matrix mapping, 41 pairs of training samples could be enough to learn the matrix mapping (see Eqs. (8) and (10) in Section 2.2).

2.2. Matrix regression

To learn $\mathbf{L}$ and $\mathbf{R}$ in (2) from the N pairs of image patches in $\mathcal{X}$ and $\mathcal{Y}$, we employ the criterion of Bayesian maximum a posteriori estimation. Under the Gaussian noise assumption, this turns out