Pattern Recognition 47 (2014) 3870–3880

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Segmentation of gray scale image based on intuitionistic fuzzy sets constructed from several membership functions

V.P. Ananthi [a], P. Balasubramaniam [a,*], C.P. Lim [b]

[a] Department of Mathematics, Gandhigram Rural Institute - Deemed University, Gandhigram 624302, Tamil Nadu, India
[b] Center for Intelligent Systems Research, Geelong Waurn Ponds Campus, Deakin University, Australia

ARTICLE INFO

Article history:
Received 5 December 2013
Received in revised form
3 June 2014
Accepted 2 July 2014
Available online 11 July 2014

Keywords:
Membership function
Thresholding
Intuitionistic fuzzy set
Hesitation degree

ABSTRACT

Segmentation is the process of extraction of objects from an image. This paper proposes a new algorithm to construct intuitionistic fuzzy set (IFS) from multiple fuzzy sets as an application to image segmentation. Hesitation degree in IFS is formulated as the degree of ignorance (due to the lack of knowledge) to determine whether the chosen membership function is best for image segmentation. By minimizing entropy of IFS generated from various fuzzy sets, an image is thresholded. Experimental results are provided to show the effectiveness of the proposed method.

1. Introduction

Segmentation is a basic and essential part in image analysis. It is a process of partitioning an image into meaningful regions. Various schemes are available to solve segmentation problems, namely gray-level thresholding, edge-detection, histogram thresholding, region, clustering algorithm, texture and so on (for more techniques, see [1]). Over last decades many segmentation techniques have been introduced in the literature [2–7], each of them is based on certain methodology to segment the regions in an image.

Usually global thresholding is utilized for segmenting images based on their gray levels. An important reason behind gray-level image segmentation is that pixels of same levels are related to a particular object in an image. Among the available segmentation methods, gray level thresholding is very simple and more efficient algorithm to classify foreground (object) from the background. Many thresholding algorithms are available in the literature (for further details, see [8–12]). But none of these algorithms threshold an image with imprecise data. After the introduction of fuzzy sets (FSs) by Zadeh [13], fuzzy set theory is used to solve segmentation problems regarding imprecise images. Various fuzzy thresholding techniques have been emerged to remove segmentation problems in imprecise images. Nowadays, FSs have been successfully used in many areas such as image processing, pattern recognition and machine learning. Main idea behind fuzzy thresholding is to first fuzzify a selected image feature by means of a suitable membership function and then to select and optimize a global or local fuzzy measure to achieve the goal of image segmentation.

In 1970, fuzzy segmentation is suggested and the results have been obtained in the form of fuzzy sets rather than crisp sets [14]. Pal et al. [8] have initiated the fuzzification process for image segmentation by selecting an appropriate membership function by optimizing a local fuzzy measure. Therefore, one must prefer the membership function that interprets the image best for good segmentation. Various membership functions have been introduced in [14–20]. *S*-function is used as the membership function for the first time in fuzzy image thresholding [8]. After this, Huang and Wang [16] have suggested various membership functions to improve image representation and this shows a better result than *S*-function based membership function. At last, Bustince et al. [14] have developed a generalization method for building membership function which is utilized in a fuzzy thresholding algorithm. These membership functions are built in such a way to relate the intensity of a pixel and an average intensity of foreground and background. Numerous researchers have been working on the segmentation of images based on fuzzy sets, for more details see [21–23]. But one of the main issues in representing an image by means of fuzzy sets is the selection of membership function which correctly portrays the information/data in the image. It is tedious to choose a membership function from the number of membership function available in the literature that perfectly reflects the particular image.

The fitness of a membership function will probably depend upon the specific characteristics of each image. Usually, the

* Corresponding author.
E-mail addresses: ananthi.gasc89@gmail.com (V.P. Ananthi), balugru@gmail.com (P. Balasubramaniam), chee.lim@deakin.edu.au (C.P. Lim).

http://dx.doi.org/10.1016/j.patcog.2014.07.003
0031-3203/

experts use a single fuzzy set (membership function) to represent the whole image. One of the problems of this kind of approaches lies on the users uncertainty when assigning the pixels either to the background or to the object through the choice of the membership function. Another issue arises in selecting peculiar membership function from this choice, which is important for the performance of an algorithm. Hence from the above discussions, there is a hesitation in the selection of membership function, which emerges due to the lack of knowledge/ignorance of the user from the choice of membership function. But none of the fuzzy algorithms concentrated on the second issue while thresholding. In order to overcome these drawbacks, IFSs are utilized in this paper for thresholding an image. This approach uses an intuitionistic fuzzy index for representing the uncertainty of the user in determining whether the chosen membership function is the best for image segmentation from the choice of membership functions. That is, if the user is absolute about the membership function, then the value of intuitionistic fuzzy index is zero (a fuzzy set). On the other hand, if the user does not know the membership degree of that pixel at all, then the value of intuitionistic fuzzy index is possibly maximum. Chaira [24] has proved that Intuitionistic Fuzzy Sets (IFSs) are capable of removing uncertainties in greater extent than fuzzy sets. Here, the uncertainty is in the selection of membership function and is represented as the intuitionistic fuzzy index in IFS. The main aim of this paper is to provide a method to handle the uncertainty in the construction of corresponding fuzzy set.

In this paper, a new method is proposed for thresholding gray scale images based on IFS constructed from several fuzzy sets. Initially, one can opt various different membership functions from which an IFS is generated with intuitionistic fuzzy index representing the uncertainty regarding the selection of membership function. Then, fuzzy sets for each L level are constructed by utilizing those selected membership functions. Then, hesitation degree is calculated for each of these fuzzy sets. Later, L-IFSs are formulated and entropy of each set is found separately. Finally, one can opt the best threshold by selecting the intensity level associated with an IFS having minimum entropy.

The framework of this paper is described as follows. Firstly, a few preliminary concepts of fuzzy sets and IFSs are presented in Section 2. Section 3 describes the basic concepts of fuzzy thresholding along with the proposed IFS algorithm. Section 4 shows some evaluation metrics for calculating performance of the algorithm. Section 5 provides experimental results and discussions. Finally, conclusion is drawn in Section 6.

2. Preliminaries

This section introduces some basic ideas of fuzzy set (FS) and IFS which is utilized throughout this paper.

2.1. Fuzzy sets

Fuzzy set theory was first proposed by Zadeh [13] in 1965. A fuzzy set F in a finite set X with cardinality M is mathematically written as

$$F=\{(x,\mu_F(x))|x\in X\},$$

where the function $\mu_F(x):X\to[0,1]$ represents the membership degree of an element x in X. Therefore, the non-membership degree of x is $1-\mu_F(x)$.

Triangular norm (t-norm) is a commutative, associative and non-decreasing function T from $[0,1]\times[0,1]$ into $[0,1]$ with $T(\mu_F(x),1)=\mu_F(x)$. Similarly, s-norm (t-conorm) is also a commutative, associative and non-decreasing function S from $[0,1]\times[0,1]$ to $[0,1]$ with $S(\mu_F(x),0)=\mu_F(x)$. Fuzzy negation n is a decreasing function from $[0,1]$ to $[0,1]$ such that $n(0)=1$ and $n(1)=0$. A strong negation is a decreasing involution from $[0,1]$ to $[0,1]$, that is $n(n(x))=x$.

An automorphism of the interval $[p,q]\subset\mathbb{R}$ is a continuous, strictly non-decreasing function $\varphi:[p,q]\to[p,q]$ with $\varphi(p)=p$ and $\varphi(q)=q$.

If φ_1 and φ_2 are two automorphisms in a unit interval, then $REF(p,q)=\varphi_1^{-1}(1-|(\varphi_2(p)-\varphi_2(q))|)$ with $n(p)=\varphi_2^{-1}(1-\varphi_2(p))$ is a restricted equivalence function (REF), where n is a strong negation and REF is a function defined as follows.

A function $REF:[0,1]^2\to[0,1]$ is said to be restricted equivalence function if it meets the following considerations (for more details, see [25]):

1. $REF(p,q)=REF(q,p)$, for all $p,\ q\in[0,1]$.
2. $REF(p,q)=1\Leftrightarrow p=q$.
3. $REF(p,q)=0\Leftrightarrow p=1,q=0$ (or) $p=0,q=1$.
4. $REF(p,q)=REF(n(p),n(q))$, for all $p,\ q\in[0,1]$, where n is a strong negation.
5. For all $p,\ q,\ r\in[0,1]$ if $p\le q\le r$, then $REF(p,q)\ge REF(p,r)$ and $REF(q,r)\ge REF(p,r)$.

REF can be constructed using automorphisms. For example, consider automorphisms $\varphi_1(p)=\ln[p(e-1)+1]$, where $e=\exp(1)$ and $\varphi_2(p)=p$. Then

$$\begin{aligned}REF(p,q)&=\varphi_1^{-1}(1-|(\varphi_2(p)-\varphi_2(q))|)\\&=\varphi_1^{-1}(1-|p-q|)\\&=(e^{1-|p-q|}-1)/(e-1)\\&=0.582(e^{1-|p-q|})-1.\end{aligned}$$

Based on FSs, an image I of $S\times T$ dimension with L gray levels can be considered as an $S\times T$ array of fuzzy singletons referring the brightness values of the pixels. Fuzzy singletons are the fuzzy sets whose support is a single point, where the support of a fuzzy set is defined as $Supp(F)=\{x\in X|\mu_F(x)>0\}$. Thus, a fuzzy set ($F$) of an image ($I$) can be defined as a function described by

$$F:S\times T\overset{I(i,j)=g}{\longrightarrow}G\overset{\mu_F(g)}{\longrightarrow}[0,1]$$

where $I(i,j)$ is (i,j)th pixel value of the image I, $I(i,j)=g\in G|G=\{0,\ 1,\ 2,\dots,L-1\}$. $\mu_F(g)$ denotes the belongingness of the element g in the image I (as defined in Section 3.2).

Hence, the image I in the fuzzy domain is defined by

$$F=\{(I(i,j),\mu_F(I(i,j)))|0\le i\le S-1,\ 0\le j\le T-1,\ 0\le I(i,j)\le L-1,\ 0\le\mu_F\le 1\}.$$

For more details about the construction of FS and IFS the reader can refer to [26].

2.2. Intuitionistic fuzzy sets

Atanassov [27] and Atanassov and Stoeva [28] have generalized fuzzy sets as IFS. An IFS F in X can be mathematically symbolized as

$$IFS=\{(x,\mu_{IFS}(x),\gamma_{IFS}(x))|x\in X\},$$

where the functions $\mu_{IFS}(x)$ and $\nu_{IFS}(x):X\to[0,1]$ represent the degree of membership and non-membership of an element x in X, respectively, with the essential condition $0\le\mu_{IFS}(x)+\nu_{IFS}(x)\le 1$.

A new parameter $\pi_F(x)$, which originates due to the lack of knowledge called hesitation degree, has been introduced by Atanassov and Stoeva [28] while computing the distance between