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Orthogonal moments based on exponent functions: Exponent-Fourier moments

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ABSTRACT

In this paper, we propose a new set of orthogonal moments based on Exponent functions, named Exponent-Fourier moments (EFMs), which are suitable for image analysis and rotation invariant pattern recognition. Compared with Zernike polynomials of the same degree, the new radial functions have more zeros, and these zeros are evenly distributed, this property make EFMs have strong ability in describing image. Unlike Zernike moments, the kernel of computation of EFMs is extremely simple. Theoretical and experimental results show that Exponent-Fourier moments perform very well in terms of image reconstruction capability and invariant recognition accuracy in noise-free, noisy and smooth distortion conditions. The Exponent-Fourier moments can be thought of as generalized orthogonal complex moments.

1. Introduction

Image moments and their nonlinear combinations have been used as global features in a variety of applications in image analysis, such as visual pattern recognition [1,2], template matching [3], object classification [4], watermarking [5], robot sensing techniques [6], and edge detection [7], because they are able to remain invariant in describing an image that has been translated, rotated and scaled. Among the different types of moments, geometric moments [8] and their extensions have played important roles in image analysis. However, these moments are not orthogonal. Consequently, the reconstruction of image from these moments is quite difficult. Moreover, it has a certain degree of information redundancy and sensitivity to noise [9].

Teague [10] suggested that orthogonal moments based on orthogonal polynomials to overcome the problems associated with geometric moments, such as Zernike and Legendre moments based on Zernike polynomials and Legendre polynomials respectively. Zernike moments (ZMs) and Legendre moments are orthogonal so that they are able to store information with minimal information redundancy and have the property of being rotation invariant. It has been widely used in character recognition [11] and image watermarking [12]. Teh et al. [13] and Mukundan et al. [14] studied the image representation capability, information redundancy, noise sensitivity, and proved that the orthogonal image moments perform better than geometric moments, rotational moments and complex moments in image representation and anti-noise capability, and have less information redundancy.

In this way, Sheng et al. proposed orthogonal Fourier–Mellin moments (OFMs) [15] based on a set of radial polynomials, and proved that orthogonal Fourier–Mellin moments have better performance than Zernike moments in terms of image reconstruction capability and noise sensitivity, especially in describing small image. And then, Zi-liang Ping et al. proposed Chebyshev–Fourier moments (CFMs) [16] based on Chebyshev polynomials and Jacobi–Fourier moments (JFMs) [17] based on Jacobi polynomials. They also pointed out that Jacobi–Fourier moments are the general form of the image moments based on polynomials. The variation of two parameters in Jacobi polynomials, α and β, can form various types of orthogonal moments: Legendre–Fourier moments ($\alpha=1,\ \beta=1$); Chebyshev–Fourier moments ($\alpha=2,\ \beta=3/2$); Orthogonal Fourier–Mellin moments ($\alpha=2,\ \beta=2$); Zernike moments and pseudo-Zernike moments, and so on. In 2003, Haiping Ren proposed Radial harmonic Fourier moments (RHFMs) based on the trigonometric functions [18]. In 2010, Bin Xiao constructed Bessel–Fourier moments (BFMs) [19] using the Bessel polynomials of the first kinds. And Pew-Thian Yap et al. proposed Polar Harmonic Transforms (PHT) [20] which include Polar Complex Exponential Transforms (PCET) based on Exponent functions, Polar Cosine Transforms (PCT) based on Cosine functions and Polar Sine Transforms (PST) based on Sine functions. So far, Polar Harmonic Transforms have the best performance overall among known orthogonal moments.

All kinds of moments based on polynomials are quite computationally expensive, because there is a great deal of exponential operation in these radial polynomials. To reduce the computational

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complexity, Yang et al. proposed an algorithm about the fast computation of Legendre moments [21], Papakostas and Chandan Singh et al. presented the efficient algorithm for the computation of the orthogonal Fourier–Mellin moments [22] and Zernike moments [23] respectively. In spite of this, their methods still need a great deal of computation, which costs a lot of CPU time. Ping et al. pointed out that the radial functions can be arbitrary functions if the integral of the moment's calculation is performed in the interval from zero to the scale factor [16]. This conclusion implies that it is possible to choose proper radial functions other than orthogonal polynomials to construct new moments. In this way, the exponent function e^x can be chosen as radial function to construct new moments. We can compute the radial function and angular factor as a united one, for the radial functions can be incorporated into the angular factors in the new moments. Therefore, computational complexity of the new moments is far less than ZMs, BFMs and RHFMs. In this paper, these new orthogonal moments are called Exponent-Fourier moments (EFMs) which are similar to RHFMs and PCET. In polar coordinate system, rotating the image does not change the magnitudes of its Exponent-Fourier moments, they are rotation invariants. Similar to other orthogonal moments based on polynomials, it is easy for image reconstruction and rotation invariant recognition. In this paper, comparison with Zernike moments, BFMs, RHFMs and PCET is provided.

The idea of constructing orthogonal image moments based on complex exponent function is proposed for the first time by the first author in his doctoral thesis [24]. Jiang [25] applied the Fast Fourier Transform (FFT) in the computation of EFMs, which makes the computation more accurate and fast.

The paper is organized as follows. In Section 2, the definitions of ZMs, BFMs, RHFMs and PCET are given. In Section 3, definitions of Exponent-Fourier moments and Exponent-Fourier moments invariants are presented. Section 4 discusses the properties and performance comparison of EFMs, BFMs, ZMs, RHFMs and PCET. In Section 5, the comparative analysis of the proposed approach with BFMs, ZMs, RHFMs and PCET in terms of the image reconstruction capability, recognition accuracy and computational load is provided. Section 6 concludes the paper.

2. Zernike moments and Bessel–Fourier moments

In this section, we review the definitions of the Zernike moments and Bessel–Fourier moments.

2.1. Zernike moments

The Zernike moments of order n with repetition m for a digital image $f(r,\theta)$ function are defined as

$$Z_{nm}=\frac{n+1}{\pi}\int_0^{2\pi}\int_0^1 [V_{nm}(r,\theta)]^* f(r,\theta) r\,dr\,d\theta \tag{1}$$

$$V_{nm}(r,\theta)=R_{nm}(r)e^{im\theta} \tag{2}$$

$$R_{nm}(r)=\sum_{s=0}^{(n-|m|)/2}(-1)^s\frac{(n-s)!}{s!((n+|m|)/2-s)!((n-|m|)/2-s)!}r^{n-2s} \tag{3}$$

where $[V_{nm}(r,\theta)]^*$ is the complex conjugate of $[V_{nm}(r,\theta)]$.

2.2. Bessel–Fourier moments

The Bessel–Fourier moments based on the Bessel function of the first kind in the polar coordinate are defined as

$$B_{nm}=\frac{1}{2\pi a_n}\int_0^{2\pi}\int_0^1 f(r,\theta)J_v(\lambda_n r)\exp(-jm\theta) r\,dr\,d\theta \tag{4}$$

where $f(r,\theta)$ is the image and $n=0,\ 1,\ 2,\ldots,\ m=0,\ \pm1,\ \pm2,\ \pm3\ldots$ are the moment orders, $a_n=[J_{v+1}(\lambda_n)^2]/2$, $J_v(\lambda_n r)$ is the Bessel polynomial in r of degree n, λ_n is the n-th zero of $J_v(r)$.

2.3. Radial harmonic Fourier moments

The definition of Radial harmonic Fourier moments is as follows:

$$\varphi_{nm}=\int_0^{2\pi}\int_0^1 f(r,\theta)H_n(r)\exp(-jm\theta) r\,dr\,d\theta \tag{5}$$

where

$$H_n(r)=\begin{cases}\dfrac{1}{\sqrt{r}} & if\ \ n=0\\ \sqrt{\frac{2}{r}}\ \sin\ [(n+1)\pi r] & if\ \ n=odd\\ \sqrt{\frac{2}{r}}\ \cos\,(n\pi r) & if\ \ n=even\end{cases} \tag{6}$$

where $f(r,\theta)$ is the image and $n=0,\ 1,\ 2,\ldots,$ $m=0,\ \pm1,\ \pm2,\ \pm3\ldots$ are the moment orders.

2.4. Polar Complex Exponential Transform

The definition of the Polar Complex Exponential Transform is as follows:

$$M_{nl}=\frac{1}{\pi}\int_0^{2\pi}\int_0^1 f(r,\theta)Q_n(n)\exp(-jm\theta) r\,dr\,d\theta \tag{7}$$

where

$$Q_n=\exp(-j\,2n\pi r^2) \tag{8}$$

$f(r,\theta)$ is the image and $n=0,\ \pm1,\ \pm2,\ldots,\ m=0,\ \pm1,\ \pm2,\ \pm3\ldots$ are the moment orders.

3. Exponent-Fourier moments

Exponent-Fourier moments are a set of moments based on the Exponent function. In this section, definition of the radial function of Exponent-Fourier moments is provided, and Exponent-Fourier moments and Exponent-Fourier moments invariant are introduced.

3.1. The radial function of the Exponent-Fourier moments

An orthogonal function set $P_{nm}(r,\theta)$ is defined in a polar coordinate system contains the radial function $T_n(r)$ and the Fourier function $\exp(jm\theta)$:

$$P_{nm}(r,\theta)=T_n(r)\exp(jm\theta) \tag{9}$$

where

$$T_n(r)=A_n\exp(jnr) \tag{10}$$

To make $P_{nm}(r,\theta)$ are orthogonal over the interval $0\le r\le1$, $0\le\theta\le2\pi$, namely

$$\int_0^{2\pi}\int_0^1 P_{nm}(r,\theta)P_{kl}^*(r,\theta) r\,dr\,d\theta=2\pi\delta_{nmkl} \tag{11}$$

Eq. (12) must be true:

$$\int_0^1 T_n(r)T_k^*(r) r\,dr=\delta_{nk} \tag{12}$$

On substituting Eq. (10) into Eq. (12), we obtain Eq. (13)

$$\int_0^1 A_n\exp(jnr)A_k\exp(-jkr) r\,dr=\delta_{nk} \tag{13}$$