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A generalized morphological skeleton transform using both internal and external skeleton points

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ABSTRACT

The morphological skeleton transform (MST) is a leading morphological shape representation scheme. In the MST, a given shape is represented as the union of all the maximal disks contained in the shape. The concept of external skeleton points and external maximal disks has been used for shape description and characterization purposes. In this paper, we develop a generalized morphological skeleton transform that combines the concepts of internal and external maximal disks into a unified framework. In this framework, a shape is described in terms of disk components that need to be added as well as disk components that need to be removed. The procedures and formulae describing the extraction of the disk components and the reconstruction of the original shape from these components are developed. The correctness of the procedures and formulae is established. This new framework seems to provide a more powerful and more natural way of modeling the approximation and reconstruction of binary shapes using primitive shape components.

1. Introduction

Shape representation is a fundamental problem in image analysis and computer vision [1,2]. Efficient shape representation schemes are the bases for the development of efficient algorithms for many shape-related processing tasks such as image coding [3,4], shape matching and recognition [5–12], content-based video processing [13–15], image data retrieval [16,17], and medical imaging [18].

Mathematical morphology provides a shape-based approach to image processing [19,20]. The basic morphological operations are defined in terms of the set-theoretic operations. But the effects of these basic morphological operations can be interpreted using geometric concepts of shape, size and distance. Another advantage of mathematical morphology is that it has a well-developed mathematical structure, which supports the development, analysis, and characterization of new morphological image processing algorithms.

A number of morphological shape representation algorithms have been proposed [3,4,21–32]. The morphological skeleton transform (MST) is a leading morphological shape representation scheme [21]. In the MST, a given shape is represented as the union of all the maximal disks contained in the shape. The advantages of the MST include that it has simple and intuitive mathematical characterizations as well as easy and efficient implementations. Some shape matching algorithms have been developed based on the MST [7,8].

The notion of skeleton or medial axis transform was first introduced by Blum [33]. Lantuejoul showed that the skeleton can be computed using morphological operations [34]. The term skeleton is also often used to describe thinning algorithms that preserve homotopy but do not necessarily support exact shape reconstruction [35,36]. In this paper, our focus is instead on building efficient morphological structural shape representations that allow exact as well as approximate reconstructions of the input shapes. Using the language of mathematical morphology, the extraction of our structural shape components as well as the reconstruction of the original shape from these components can all be described using well-defined formulae. Therefore, we are following a structural and algebraic approach to shape representation.

A number of morphological shape representation schemes can be viewed as variants of the MST. The morphological shape decomposition (MSD) is another important morphological shape representation scheme [22]. In the MSD, a given shape is represented as a union of certain disks contained in the shape. The overlapping among representative disks of different sizes is eliminated. Another morphological shape representation algorithm that can be viewed as a compromise between the MST and the MSD was proposed in [30]. In this scheme, overlapping among representative disks of different sizes is allowed, but severe

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overlapping among such disks is avoided. This algorithm can be called overlapped morphological shape decomposition (OMSD). It was pointed out in [30] that compared to the MSD, the MST and OMSD usually use fewer disks to represent a shape. The OMSD decomposes a shape into a number of modestly overlapped shape components. These components seem to correspond better to the natural shape parts than those generated by the MSD.

A generalized skeleton transform was introduced in [31] that derives generalized skeleton points for a given shape image using multiple structuring elements. Each such skeleton point represents a generalized maximal "disk," which in general is an octagon. It was reported in [31] that the number of skeletal points used to represent a shape is significantly lower than those used by the MST, MSD, and OMSD. The main advantage of this generalized skeleton transform is that it leads to an efficient shape decomposition scheme which also allows a shape to be approximated accurately by using only a small number of shape components. A modification to this algorithm and an associated shape matching algorithm were described in [32]. In these two schemes, a shape component is a special type of octagon whose shape is specified using 4 integers while in the MST, MSD, and OMSD, the shape of a disk component is specified using a single size number. There are situations where simpler shape components are preferred.

The efforts to improve representational efficiency of the MST also include [37–39]. A generalization to the MST was described in [39]. The use of multiple structuring elements and repeated applications of the basic skeleton transform allow higher compression rates to be achieved. A generalized fuzzy mathematical morphology and its application in object representation were presented in [37]. This fuzzy morphology allows noisy binary shapes to be represented more accurately and efficiently. However, the noisy shapes cannot be completely reconstructed. In [38], the theory of spatially variant mathematical morphology was applied to minimize the cardinality of skeleton representation. The algorithm used is search-based. More efficient representations are obtained at the cost of higher computational complexity. The more complicated relationships between the skeleton points determined and the original shape may limit its applications.

A great deal of research has been done in the area of shape decomposition [40–44,47,48]. However, most works focus on decomposing a shape into simpler non-overlapping segments. Very often, these simpler segments are convex polygons or near-convex parts of the input shape. Many of these algorithms use combinations of several techniques to extract shape segments from the input shape. In the MST, a maximal disk does not always correspond to a natural shape part and there is a great deal of overlapping among the maximal disks. However, the basic ideas of the MST can be used, combined with other techniques, to develop shape decomposition algorithms.

Many skeleton-based shape matching algorithm have been developed, although they do not always use the skeleton points generated by the morphological skeleton transform directly. Shock structures [10] are skeleton points and segments with dynamic information of direction and speed of flow that comes from the interpretation of the skeleton as the locus of singularities (shocks) formed by wave propagation from the boundary. Therefore, the shock graph, built from shock structures, is a richer description than the skeleton. One weakness of the associated shape matching algorithm is its complex graph editing operations and high computational costs. The disconnected skeleton [12] is formed by identifying local symmetry points corresponding to the curvature extrema of the evolving boundary. A local symmetry axis ends as soon as the evolving curve locally becomes a circle. The disconnected axes correspond to ribbon-like sections of the shape being described. This representation does not provide a complete description of the given shape. Its main purpose is to support robust shape comparison under various deformations and transformations. The path similarity skeleton graph matching [11] is based on comparing the shortest paths between each pair of endpoints of the pruned skeletons. These shortest paths do not provide a complete description of the input shape. But they are effective shape descriptors that support efficient and robust shape matching under various deformations and transformations.

In a recent paper, a structural shape matching algorithm that uses both internal and external shape components was developed [6]. The internal shape components are selected from the internal maximal disks determined by a traditional MST. The external shape components are selected from the external maximal disks determined by a separate "external" skeleton transform. An internal shape component represents an area of the input shape, while an external shape component represents an area outside the input shape. The concept of "external" skeleton points is not new. The idea has been used in works such as [23,45].

In this paper, we develop a generalized morphological skeleton transform that combines the concepts of internal and external maximal disks into a unified framework. In this framework, a shape is described in terms of positive disk components that need to be added as well as negative disk components that need to be removed. The positive and negative disk components are extracted from the progressively simplified versions of the input shape. The step for extracting positive disk components and the step for extracting negative disk components are applied alternately to derive the final representation of the input shape. Formulae describing the extraction of the disk components as well as the reconstruction of the original shape from the extracted components are developed. The proof of correctness of these formulae is also provided. A preliminary version of the paper has been presented at a conference [46].

In Section 2, we review the concepts of internal and external skeleton transforms. These two concepts are combined into a unified framework in Section 3. In Section 4, we describe a simple shape decomposition algorithm based on the skeleton transform. A number of representation and decomposition examples are given in Section 5. Some generalizations of the basic representation algorithm are described in Section 6. Section 7 gives conclusions.

2. Internal and external skeleton transforms

We first review the standard skeleton transform. We use the definitions for basic morphological operations given in [20]. For a structuring element B, which is used as the unit disk, we define size-i disk iB as

$$iB = B \oplus B \oplus \ldots B \ (i \text{ copies of } B). \tag{1}$$

Since the opening operation is anti-extensive, for a shape image X, we can write

$$X = (X \circ B) \cup S_0 = ((X \ominus B) \oplus B) \cup S_0 = (X_1 \oplus B) \cup S_0 \tag{2}$$

where

$$S_0 = X \setminus (X \circ B) \tag{3}$$

$$X_1 = X \ominus B. \tag{4}$$

For X_1 in (2), we can also write

$$X_1 = (X_1 \circ B) \cup S_1 = ((X_1 \ominus B) \oplus B) \cup S_1 = (X_2 \oplus B) \cup S_1 \tag{5}$$

where

$$S_1 = X_1 \setminus (X_1 \circ B) = (X \ominus B) \setminus ((X \ominus B) \circ B) \tag{6}$$

$$X_2 = X_1 \ominus B = X \ominus 2B. \tag{7}$$

Combining (2) with (5), we get