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Moments and moment invariants in the Radon space

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ABSTRACT

Radon transform has been acknowledged as the promising solution for image processing due to its high noise robustness and the ability of converting the rotation, scaling and translation operations on a pattern image into translations and scaling in the Radon image. Recently, several transforms widely employed in signal processing have been introduced in images' Radon space for pattern recognition. However, moments and especially moment invariants in the Radon space have not been thoroughly investigated. In this paper, we introduce a mathematical framework of constructing moments and moment invariants in the Radon space. First, rotational moments which represent non-orthogonal moments and Legendre–Fourier moments which represent orthogonal moments are introduced in the Radon space respectively. On this basis, we propose a method to obtain rotation, scaling and translation as well as affine invariance of these moments in the Radon space. Second, we prove that the proposed moments in the Radon space can be represented by a linear combination of classical geometric moments. With this property, the implementation time of the moments in the Radon space can be significantly reduced, and the recognition accuracy can also be greatly improved since no numerical approximation is involved. Theoretical and experimental analysis on invariant recognition accuracy, noise robustness, image blur distortion and computational time also shows the superiority of the proposed methods.

1. Introduction

Recognition of objects when subjected by various geometric distortions in noisy and cluttered environments is an important challenge in pattern recognition and computer vision. Invariant feature extraction is the most popular method to this problem [1,2]. While moments are scalar quantities used to characterize a function and to capture its significant feature, in the past few years, the use of moments and moment invariants has received a wide range of attention in invariant image recognition [3]. As illustrated in Flusser et al. [3], the moment methods can be categorized into non-orthogonal and orthogonal families. The family of non-orthogonal moments mainly includes geometric moments, complex moments and radial moments (or rotational moments) [4]. The family of orthogonal moments mainly includes Zernike moments [5], orthogonal Fourier–Mellin moments [6], pseudo-Zernike moments [7], Bessel–Fourier moments [8], Legendre [9], Gaussian–Hermite [10], Tchebichef [11], Krawtchouk [12], and Hahn moments [13]. Based on these moments, various moment invariants have been proposed. Hu [14] first derived seven moment invariants constructed on geometric moments to achieve rotation, scale and translation (RST) invariance. By correcting the fundamental theorem of moment invariants, Flusser and Suk [15] introduced a new set of moment invariants to images under affine transform. Correspondingly, Ghorbel et al. [16] developed a method to derive RST moment invariants from complex moments. Reddi [17] provided a framework for deriving RST moment invariants constructed on radial and angular moments. Moreover, moment invariants based on orthogonal moments have also been introduced. Chong et al. discussed the translation invariance of Zernike moments [18], the scale invariance of pseudo-Zernike moments [19] as well as the scale and translation invariance of Legendre moments [20] respectively. Based on the same theory, Zhu et al. [21] derived the translation and scale invariance of discrete Tchebichef moments. Recently, the RST invariance of orthogonal Fourier–Mellin moments [22], Gaussian–Hermite moments [23] and radial discrete Tchebichef moments [24] are deeply discussed.

In recent years, various transforms are employed after image's Radon transform for RST invariant image recognition. Jafari et al. [25,26] introduced wavelet transform in the Radon space for rotation invariant texture retrieval and proved that the signal-to-noise ratio (SNR) has increased by $1.7N_R$ after the Radon transform (where N_R is the image size). Tabbone et al. [27] provided an integral function and the discrete Fourier transform on the radial and angular coordinates of the Radon image respectively to get RST invariance. Wang and Xiao developed the 2D Fourier–Mellin transform in the Radon space for rotation and scaling invariant

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image recognition [28]. Inspired by the same theory, Hoang and Tabbone [29] applied 1D Mellin and discrete Fourier transforms on the radial and angular coordinates of the Radon image respectively to achieve RST invariance.

The pioneer work of estimation moments in the Radon space was represented by Hiriyannaiah and Ramakrishnan [40], who obtained general and orthogonal moments from the Radon transform data as well as established the relationship between the regular moments from Radon projection data and the geometric moments. The first attempt to find RST invariance from moment patterns in the Radon space was investigated by Galigekere et al. [30]. They introduced geometric moment along r-axis of the Radon image for scale and translation invariant image recognition, but the rotation invariance is achieved by estimating the rotated angle with time consuming circular correlation methods. Zhu et al. [31] presented a watermarking scheme that can resist RST transform attack by the complex moments from images Radon projection. However, all the methods mentioned above are built on the same framework. The Radon transform is performed first, and various transforms followed. Therefore, the implementation of these methods is extremely time consuming since four or more tuple integrals are involved. Moreover, another drawback in the aforementioned methods is the discretization error and numerical approximation error, which accumulates as the order of the transforms (followed by the Radon transform) increases. It limits the accuracy of the recognition.

In this paper, we propose a framework for constructing moments and moment invariants in the Radon space. Based on this framework, we introduce two types of moment methods named rotational moments and Legendre–Fourier moments in image's Radon space for image analysis and invariant image recognition. To the best of our knowledge, it is the first time to demonstrate these two moments in the Radon space that can be represented by a linear combination of geometric moments without variable θ. Therefore, it makes the calculation of the proposed methods very simple and efficient by reducing the tuple of integrals from four to two. Moreover, thanks to the exact computation of geometric moment method, the numerical approximation errors involved in the proposed methods can be eliminated to increase the recognition accuracy. In addition, the affine invariant of the proposed methods can also be achieved by replacing the geometric moment with affine moment invariants.

The rest of this paper is organized as follows. In Section 2, we briefly review the definitions of Radon transform, geometric moments and exact geometric moments to build up fundamental mathematical background. The definition and RST invariance of rotational moments in the Radon space are introduced in Section 3. The relationship between rotational moments in the Radon space and geometric moment is also given in this section. The same properties of Legendre–Fourier moments in Radon space are discussed in Section 4. Section 5 introduces the affine invariance of rotational moments and Legendre–Fourier moments in Radon space. Experimental results and analysis are described in Section 6, and conclusions are presented in Section 7.

2. Some basic definitions

This section provides a brief review of the definitions of Radon transform, geometric moments and fast and exact geometric moments. All of these definitions will be used later in this paper.

2.1. Radon transform

Let $f(x,y)$ be an image function, the Radon transform of $f(x,y)$ can be defined as follows [41]:

$$R(r,\theta)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\cdot\delta(r-x\cos\theta-y\sin\theta)\,dx\,dy \tag{1}$$

where $\delta\{\cdot\}$ denotes the Dirac delta function, r is the perpendicular distance of a straight line from the origin, and θ is the angle between the distance vector and the x-axis, i.e., $\theta\in[0,\pi)$.

2.2. Fast and exact geometric moments

The $p+q$ order geometric moments of image $f(x,y)$ are defined as

$$M_{p,q}=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)x^p y^q\,dx\,dy \tag{2}$$

To avoid the numerical approximation errors involved in the computation of geometric moments, Hosny [32] introduced the exact 2D geometric moments that are defined as follows:

$$M_{p,q}=\sum_{i=1}^{M}\sum_{j=1}^{N} h_{p,q}(x_i,y_j)f(x_i,y_j) \tag{3}$$

where M,N is the image size, the kernel function $h_{p,q}(x_i,y_j)$ can be written as

$$\begin{aligned} h_{p,q}(x,y)&=\int_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2}\int_{y_i-\Delta y_i/2}^{y_i+\Delta y_i/2} x^p y^q\,dx\,dy\\ &=\frac{1}{p+1}x^{p+1}\Big|_{x_i-\Delta x_i/2}^{x_i+\Delta x_i/2}\cdot\frac{1}{p+1}y^{q+1}\Big|_{y_i-\Delta y_i/2}^{y_i+\Delta y_i/2}\\ &=\left(\frac{1}{p+1}(-1+i\Delta x_i)^{p+1}-(-1+(i-1)\Delta x_i)^{p+1}\right)\\ &\quad\cdot\left(\frac{1}{q+1}(-1+j\Delta y_j)^{q+1}-(-1+(j-1)\Delta y_j)^{q+1}\right) \end{aligned} \tag{4}$$

In the above equation, the intervals Δx_i and Δy_i are fixed at constant values $\Delta x_i=2/M$ and $\Delta y_i=2/N$ respectively.

Similar to the method of Fourier transform, the time complexity of 2D $(p+q)$-order geometric moments can be significantly reduced by successive computation of the 1D q-order moments for each row:

$$M_{p,q}=\sum_{i=1}^{M}\left(\frac{1}{p+1}(-1+i\Delta x_i)^{p+1}-(-1+(i-1)\Delta x_i)^{p+1}\right)Y_{i,q} \tag{5}$$

where

$$Y_{i,q}=\sum_{j=1}^{N}\left(\frac{1}{q+1}(-1+j\Delta y_j)^{q+1}-(-1+(j-1)\Delta y_j)^{q+1}\right)f(x_i,y_j) \tag{6}$$

3. Rotational moments in the Radon space

3.1. The definition of rotational moments in the Radon space

The Fourier–Mellin and Radon transform by applying 2D Fourier–Mellin transform in image's Radon space (r,θ) is defined as follows [28]:

$$D_{s,m}=\int_0^1\int_0^{2\pi} R(r,\theta)r^{\sigma-ju-1}e^{-jm\theta}\,dr\,d\theta \tag{7}$$

where $j=\sqrt{-1}$, $R(r,\theta)$ is the Radon transform of image function $f(x,y)$ and $m=0,\pm1,\pm2,\ldots$ is the circular harmonic order. According to the definition, the order of Mellin transform $s=\sigma-ju$ $(u\in R)$ is a complex value with σ being a fixed and strictly positive real constant. Let u=0 and σ be a non-negative integer, thus, we can define the rotational moments also called radial moments in the Radon space (RMRs) as

$$\begin{aligned} F_{n,m}&=\int_0^1\int_0^{2\pi} R(r,\theta)r^n e^{-jm\theta}\,dr\,d\theta\\ &=\int_0^1\int_0^{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\cdot\delta(r-x\cos\theta \end{aligned}$$