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Generalized dual Hahn moment invariants

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ABSTRACT

In this work we introduce a generalized expression of the weighted dual Hahn moment invariants up to any order and for any value of their parameters. In order for the proposed invariants to be formed, the weighted dual Hahn moments (up to any order and for any value of their parameters) are expressed as a linear combination of geometric ones. For this reason a formula expressing the nth degree dual Hahn polynomial, for any value of its parameters, as a linear combination of monomials ($c_r \cdot x^r$), is proved. In addition, a recurrent relation for the fast computation of the aforementioned monomials coefficients (c_r) is also given. Moreover, normalization aspects of the generalized weighted dual Hahn moment invariants are discussed, while a modification of them is proposed in order to avoid their numerical instabilities. Finally, experimental results and classification scenarios, including datasets of natural scenes, evaluate the proposed methodology.

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1. Introduction

Image moments have been attracting the interest of the scientific community for many years due to their capability of representing image information in a compact way. This ability encourages scientists to apply them in the research fields of image processing [1–5] and pattern recognition [6,7] with great results. The introduction of moments in image analysis has aroused great interest in other research directions. These directions, according to Papakostas et al. [8], involve: (1) the acceleration of moments computation, (2) the improvement of moments accuracy and (3) the incorporation of invariant properties into the moments.

Traditionally, the moments are divided into two major categories: (1) the non-orthogonal and (2) the orthogonal ones. The geometric moments are the most representative family of the first category and due to their simple implementation constitute the first type of moments introduced. However, they suffer from high information redundancy due to the lack of orthogonality and thus, they are characterized by lack of efficiency in difficult problems where more discriminative information needs to be captured. Additionally, the first introduced moment invariants [9], a set of seven invariant moments which have the ability to remain unchanged under different kinds of transformations in image plane (rotation, scaling and translation), are based on geometric, central and normalized image moments, and thus they inherit the same problem that the geometric ones have.

The aforementioned drawback motivated scientists to develop the second category of moments, the orthogonal ones, which use as kernel functions polynomials that constitute an orthogonal basis. The property of orthogonality provides the corresponding moments with the feature of minimum information redundancy, meaning that different moment orders describe different image content. The orthogonal moments can be further categorized in continuous [10] and discrete [11] ones according to the kind of their basis function. The fact that the continuous orthogonal moments are defined in a continuous space causes approximation errors and imposes the transformation of the image coordinates in order for moment families such as Legendre or Zernike to be defined. In contrast, discrete orthogonal moment families like Tchebichef [11], Krawtchouk [12], dual Hahn [13] and Racah [14] are free of the aforementioned problems, since they are natively defined in a discrete space.

Zhu et al. [15] showed that the weighted dual Hahn moments (WHMs), up to a specific order and for zero values of their polynomial parameters, can be expressed in geometric terms. They also introduced an invariant feature vector of weighted dual Hahn moment invariants (WHMIs) based on geometric moment invariants. However, since the WHMIs were first introduced,

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no generalized form for calculating these moments for any order as well as for any value of their polynomial parameters has been presented.

This work aims to extend the computation of the WHMIs for any order and for any value of their polynomials parameters. In the next pages of this work, this extended form is going to be referred to as generalized weighted dual Hahn moment invariants (GWHMIs). Another aspect that is going to be discussed regarding the WHMIs, is their incapability to be computed for large images due to overflow issues. This disadvantage is inherited by the proposed generalized ones and thus, a modification of the GWHMIs is also proposed in order to overcome this shortcoming. From this point on, the modified GWHMIs are going to be referred to as generalized dual Hahn moment invariants (GHMIs).

To sum up, there are two distinctive goals. The first one is to develop a formula able to extend Zhu's weighted dual Hahn moment invariants [15] to high order moments and for any value of their parameters. The second goal is relative to the modification of this formula in order to be free of overflow issues. These two targets, especially the first one, are very important for the following reasons: (1) Based on the fact that different moment orders describe different kinds of image content (low moment orders describe the coarse information, whereas higher orders are relative to the detailed information of image's content), the capability of the GWHMIs and GHMIs to be computed up to any order, offers the ability of representing the "most useful" image information for a specific problem. (2) The ability to change the values of the dual Hahn moments parameters improves their discriminative power, since, these parameters are relative to the property of the corresponding moments to focus on local image areas. Thus, a proper selection of the moments parameters can lead the proposed invariants to focus mainly on the critical information of image content.

The paper is organized as follows. Section 2 describes the basic theory of dual Hahn moments. Section 3 presents the proposed methodology and a discussion on normalization aspects of the WHMIs and GWHMIs. Section 4 describes how the dual Hahn parameters can be selected. Section 5 presents the experimental results and finally Section 6 includes the conclusions of this work.

2. Dual Hahn moments

This section aims to present a short introduction to the theoretical background of dual Hahn moments. Basic concepts of dual and weighted dual Hahn polynomials followed by the corresponding moments and their moment invariants are presented.

2.1. Dual Hahn polynomials

The nth order dual Hahn polynomials, defined on a non-uniform lattice, are given according to [15] by

$$W_n^{(c)}(s,a,b) = R_n^{(c)}(a,b)\,{}_3F_2(-n,a-s,a+s+1;a-b+1,a+c+1;1)$$

$$n \in \{0,\mathbb{Z}^*\},\quad a \le s \le b-1$$

$$-1/2 < a < b,\quad |c| < 1+a,\quad b = a+N \tag{2.1}$$

where the parameters a and c affect the shape of the dual Hahn polynomials

$$R_n^{(c)}(a,b) = \frac{(a-b+1)_n (a+c+1)_n}{n!} \tag{2.2}$$

and $(u)_k$ is the Pochhammer symbol defined as

$$(u)_k = u(u+1)\cdots(u+k-1),\quad k \ge 0,\quad (u)_0 = 1 \tag{2.3}$$

${}_3F_2(\cdot)$ is the generalized hypergeometric function defined as

$${}_3F_2(a_1,a_2,a_3;b_1,b_2;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k}{(b_1)_k (b_2)_k} \frac{z^k}{k!} \tag{2.4}$$

Due to the fact that the range of values of the dual Hahn polynomials expands as the order increases, Zhu et al. [15] introduced a set of weighted dual Hahn polynomials in order for the corresponding moments to be properly defined.

2.2. Weighted dual Hahn polynomials

The dual Hahn polynomials are scaled by a factor in order to achieve numerical stability. These weighted polynomials are given by

$$\tilde{W}_n^{(c)}(s,a,b) = W_n^{(c)}(s,a,b)\sqrt{\frac{\rho(s)}{d_n^2}\left[\Delta x\left(s-\frac{1}{2}\right)\right]}$$

$$n = 0,1,\ldots,N-1 \tag{2.5}$$

where

$$\rho(s) = \frac{\Gamma(a+s+1)\Gamma(c+s+1)}{\Gamma(s-a+1)\Gamma(b-s)\Gamma(b+s+1)\Gamma(s-c+1)} \tag{2.6}$$

$$d_n^2 = \frac{\Gamma(a+c+n+1)}{n!(b-a-n-1)!\Gamma(b-c-n)} \tag{2.7}$$

$$\Delta x(s) = x(s+1) - x(s) \tag{2.8}$$

$$x(s) = s(s+1) \tag{2.9}$$

and $\Gamma(z)$, $Z \in \mathbb{C}$ is the gamma function which is analytic everywhere except for the non-positive integers. Supposing that $n = 1,2,3,\ldots$, then we can write

$$\Gamma(n+1) = n!,\quad \Gamma(1) = \Gamma(2) = 1 \tag{2.10}$$

2.3. Weighted dual Hahn moments

The WHMs can be formed by using the weighted dual Hahn polynomials as a kernel. Supposing that we have an $N \times N$ image with intensity function $f(x,y)$, then the $(n+m)$ th order WHMs are given by

$$\tilde{H}_{nm} = \sum_{s=a}^{b-1}\sum_{t=a}^{b-1} \tilde{W}_n^{(c)}(s,a,b)\tilde{W}_m^{(c)}(t,a,b) f(s-a,t-a)$$

$$n,m = 0,1,\ldots,N-1 \tag{2.11}$$

where the parameters n and m correspond to the degrees of the polynomials which are used as basis functions and determine the moment order.

2.4. Derivation of invariants using geometric moments

According to [15], in order to obtain the translation, scale and rotation invariants of the WHMs, the process which is described in [12] can be adopted. In this work, Yap et al. based on a specific image transformation and using the geometric moments, succeeded in expressing the Krawtchouk moments in geometric terms. Following the same technique, Zhu et al. show that the WHMs of the transformed image $\tilde{f}_{st}(x,y) = [\rho(s)\rho(t)(2s+1)(2t+1)]^{-1/2} f(x,y)$ can be written as

$$\tilde{H}_{nm} = \sum_{s=a}^{b-1}\sum_{t=a}^{b-1} \tilde{W}_n^{(c)}(s,a,b)\tilde{W}_m^{(c)}(t,a,b)\tilde{f}_{st}(s-a,t-a)$$

$$= (d_n d_m)^{-1} \sum_{s=a}^{b-1}\sum_{t=a}^{b-1} W_n^{(c)}(s,a,b) W_m^{(c)}(t,a,b) f(s-a,t-a) \tag{2.12}$$

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