Pattern Recognition 43 (2010) 3693–3698

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Wrap-around effect removal finite ridgelet transform for multiscale image denoising

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ARTICLE INFO

Article history:
Received 29 October 2009
Received in revised form
21 April 2010
Accepted 25 May 2010

Keywords:
Finite radon transform
Ridgelets
Pyramid
Image denoising

ABSTRACT

Denoising in wavelet domain is an attractive research field. Recently, the research focused on finding some algorithms that can detect line singularities of image while smoothing noise. Based on the finite Radon transform a finite ridgelet transform was derived for this purpose. But the wrap-around effect of the finite Radon transform limits its power greatly. In this paper, a way is pointed out to remove the wrap-around effect of the finite Radon transform. By embedding it into a moving window pyramid, a multiscale image denoising algorithm is developed.

1. Introduction

Image denoising in wavelet domain has shown significant advantages. Soft thresholding [1] is the most well known method. Since wavelets in 2-D are often obtained by a tensor product of 1-D wavelets, the lines in an image cannot be detected effectively. In his pioneering work, Candes [2] developed a new system of representations called ridgelets to overcome this problem. Carre and Andres [3] proposed an implementation of the ridgelet transform based on discrete analytical 2-D lines. Chen and Kegl [4] presented an image denoising method by incorporating the dual-tree complex wavelets into the ordinary ridgelet transform.

However, the discrete version of the ridgelet transform would result in either redundancy or non-reconstruction. It is interesting that based on the finite Radon transform [5], Do and Vetterli [6] introduced a finite ridgelet transform that achieves both invertiblility and nonredundancy. Unfortunately, the wrap-around effect of the finite Radon transform limits its power in image denoising significantly [6].

In this paper, the wrap-around effect of the finite Radon transform with the size of three is removed by a simple way. Especially, based on a moving window pyramid, a multiscale image-denoising algorithm is developed. Examples show that the proposed approach is effective for preserving details of image while smoothing noise.

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doi:10.1016/j.patcog.2010.05.032

2. Finite ridgelet transform

To give the definition of the finite ridgelet transform, we need to introduce the finite Radon transform (FRT) first.

Let $Z_p=\{0, 1,\ldots, p-1\}$ where p is a prime number. The finite Radon transform (FRT) of an image $x(i,j)$, $(i,j)\in Z_p^2$, with the size of p is defined as

$$r_k[l]=\sum_{(i,j)\in L_{k,l}}[x(i,j)-y_0] \tag{1}$$

where

$$L_{k,l}=\{(i,j); j=ki+l(\text{mod}p), i\in Z_p\},\quad 0\le k<p \tag{2}$$

$$L_{p,l}=\{(l,j);\ \ j\in Z_p\} \tag{3}$$

$$y_0=\frac{1}{p^2}\sum_{(i,j)\in Z_p^2}x(i,j) \tag{4}$$

which denote the "lines" on Z_p^2 and the local mean, respectively. It is easy to show that the original image $x(i,j)$ can be reconstructed by

$$x(i,j)=\frac{1}{p}\sum_{(k,\ l)\in R_{i,j}}r_k[l]+y_0,\quad (i,j)\in Z_p^2 \tag{5}$$

where

$$R_{i,j}=\{(k,l); l=j-ki(\text{mod}\,p), i\in Z_p\}\cup\{p,i\} \tag{6}$$

In the above procedure, by applying a 1-D wavelet transform to the line projection matrix $r_k[l]$ column by column, the finite ridgelet transform (FRIT) is obtained [6].

3. Modified FRIT

Unlike the Radon transform, the FRT cannot always sum the image pixels over the set of line, which may cause the wrap-around effect. However, in the case of $p=3$, this effect can be removed by a simple way.

Fig. 1(a) shows the pixels $x(i,j)$, $0 \le i,\ j \le 2$, of a 3×3 image whose corresponding sets of pixels that make up the lines $L_{k,l}$, $0 \le k \le p$, $0 \le l < p$ are shown in Fig. 2(a)–(d). It demonstrates that the pixels in the line $L_{1,1}$ shown in Fig. 2(b) with black color are split by a diagonal of the image. Similar case happens to the lines $L_{1,2}$, $L_{2,0}$ and $L_{2,1}$.

To overcome this problem, we can swap the data in the two diagonals of the image and then do the FRT. The result is shown in Fig. 1(b), whose corresponding lines are shown in Fig. 3(a)–(d). Thus, as shown in Fig. 2(a), (d) and Fig. 3 (b), (c), the split is avoided.

Let $r_k[l]$ and $\tilde{r}_k[l]$, $0 \le k \le 3$, $0 \le l \le 2$ denote the FRT of the 3×3 images shown in Fig. 1(a) and (b), respectively. We have $\tilde{r}_0[1]=r_0[1]$, $\tilde{r}_1[0]=r_1[0]$, $\tilde{r}_2[2]=r_2[2]$ and $\tilde{r}_3[1]=r_3[1]$. Thus, from (5) the estimate of the center pixel of the image can be computed by

$$\hat{x}(1,1)=(r_0[1]+\tilde{r}_1[0]+\tilde{r}_2[2]+r_3[1])/3+y_0 \tag{7}$$

a

X(0,2)	x(1,2)	x(2,2)
x(0,1)	x(1,1)	x(2,1)
x(0,0)	x(1,0)	x(2,0)

b

X(2,0)	x(1,2)	x(0,0)
x(0,1)	x(1,1)	x(2,1)
x(2,2)	x(1,0)	x(0,2)

Fig. 1. (a) Pixels of a 3×3 image and (b) the rearranged pixels of the 3×3 image.

Then, the process, called modified FRIT (MFRIT), is described with the following steps:

1. Compute the mean y_0 of the image and then subtract it from each of the pixels within the image.
2. Apply the FRT to the pixels in the image to obtain the output $r_k[l]$, $0 \le k \le 3$, $0 \le l \le 2$.
3. Rearrange the pixels in the image by the rule shown in Fig. 2(b).
4. Apply the FRT to the rearranged image to obtain the output $\tilde{r}_k[l]$, $0 \le k \le 3$, $0 \le l \le 2$.
5. Apply the soft thresholding [1] to the project sets $\{r_0[0], r_0[1], r_0[2]\}$, $\{\tilde{r}_1[1],\ \tilde{r}_1[0],\ \tilde{r}_1[2]\}$, $\{\tilde{r}_2[0],\ \tilde{r}_2[2],\ \tilde{r}_2[1]\}$ and $\{r_3[0], r_3[1], r_3[2]\}$, respectively, to obtain the estimates $\hat{r}_0[1]$, $\hat{\tilde{r}}_1[0]$, $\hat{\tilde{r}}_2[2]$ and $\hat{r}_3[1]$.
6. Sum up the four estimates, each of which is divided by three, and the mean y_0 to make out the estimate of the center pixel of the image.

When a line has N pixels, compared with the point singularity detector the line singularity detector(ridgelet transform) can provide 10lg$N$dB signal to noise gain (SNRG). In other words, the 3×3 window based MFRIT can get at most 4.78 dB SNRG in line detecting.

The above MFRIT only gives the estimate of the center pixel of a 3×3 image. How to use it to realize multiscale image denoising is the task of the next section.

4. MFRIT for multiscale image denoising

A simple structure for 2-D multiscale representation is the window pyramid [7], which is a collection of windows with decreasing size arranged in the shape of a pyramid. Thus, by embedding the MFRIT into a moving window pyramid properly, a multiscale image denoising algorithm is set up.

In fact, suppose that the center of a $3^s\times 3^s$ filter window is at the pixel position (i,j) of a noisy image at a time. The MFRIT-based

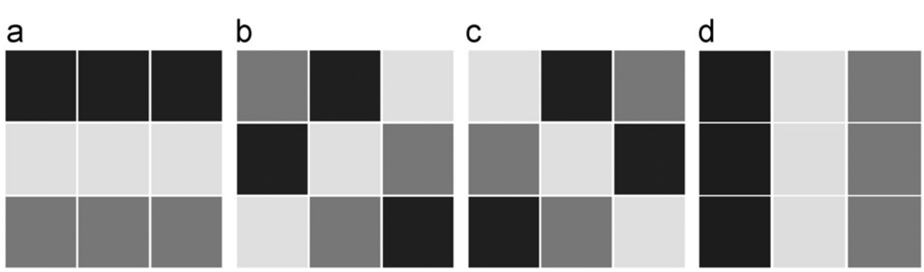

Fig. 2. The lines obtained by applying FRT to the pixels shown in Fig. 1(a): (a) lines $L_{0,l}$, $0 \le l \le 2$, (b) lines $L_{1,l}$, $0 \le l \le 2$, (c) lines $L_{2,l}$, $0 \le l \le 2$ and (d) lines $L_{3,l}$, $0 \le l \le 2$.

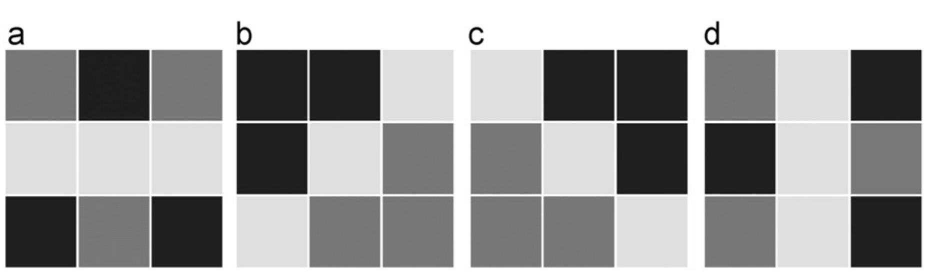

Fig. 3. Lines obtained by applying FRT to the rearranged pixels shown in Fig. 1(b): (a) lines $L_{0,l}$, $0 \le l \le 2$, (b) lines $L_{1,l}$, $0 \le l \le 2$, (c) lines $L_{2,l}$, $0 \le l \le 2$ and (d) lines $L_{3,l}$, $0 \le l \le 2$.