Pattern Recognition 43 (2010) 3805–3816

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Classifying transformation-variant attributed point patterns

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ARTICLE INFO

Article history:
Received 27 August 2009
Received in revised form
25 May 2010
Accepted 28 May 2010

Keywords:
Point pattern matching
Hausdorff distance
SAR
Multidimensional scaling
Fingerprint

ABSTRACT

This paper presents a classification approach, where a sample is represented by a set of feature vectors called an attributed point pattern. Some attributes of a point are transformational-variant, such as spatial location, while others convey some descriptive feature, such as intensity. The proposed algorithm determines a distance between point patterns by minimizing a Hausdorff-based distance over a set of transformations using a particle swarm optimization. When multiple training samples are available for each class, we implement multidimensional scaling to represent the point patterns in a low-dimensional Euclidean space for visualization and analysis. Results are demonstrated for latent fingerprints from tenprint data and civilian vehicles from circular synthetic aperture radar imagery.

1. Introduction

Suppose that a class sample is represented by a collection of feature vectors consisting of transformation-variant features along with other attributes. For example, a fingerprint can be represented by a set of minutiae, each containing two-dimensional (2D) spatial location along with type and orientation attributes [1]. The collection of feature vectors is referred to as an attributed point pattern or point set. The locations are subject to an unknown rigid transformation. When comparing two different samples, it may be necessary to register the location information before comparing the two point sets.

Researchers have approached point set classification by adopting a data representation that is invariant to a class of transformations. Example methods include graph based methods [2] and spectral correspondence methods that compare the point adjacency matrices from patterns [3,4]. Typically, the transformation invariant approaches yield matching performances that break down in the presence of noise, clutter, and occlusion [5].

Other researchers have developed methods to directly register two sets. Early methods [6,7] require a correspondence and minimize the sum of squared distances between corresponding points. Later methods iteratively discover a correspondence during the registration process with the iterative closest point (ICP) method [8,9]. To remove the correspondence all together, some have applied Hausdorff distance based registration [10,11] or a difference of convex functions formulation [12].

In this paper we estimate the registration of sets using a Hausdorff distance based technique. Registration and set distance are related; the estimated registration between two sets yields the minimum distance between the two sets. Thus we refer to the distance between two sets under their estimated registration simply as the set distance (SD). Partial versions of the Hausdorff distance [10,13,14] are naturally robust to clutter and occlusions since they find a best subset match between two sets.

In previous work, Yin [11] demonstrated a method to register two 2D point patterns by minimizing a partial Hausdorff distance between the two patterns with a particle swarm optimization. The Yin article registered synthetically generated 2D point patterns perturbed by rigid transformations, random clutter, and random occlusions. We extend Yin's contribution with four meaningful steps:

- In addition to 2D location, we include attributes that add more information to the point pattern. The associated distance between individual points can be characterized with a Mahalanobis distance using an appropriately selected error covariance matrix.
- In addition to registering point patterns, we use a version of the minimized partial Hausdorff distance as a pseudo-metric for a nearest neighbor (NN) classifier. Augmented distance matrices created from multiple samples are observed to be nearly positive definite, which reveals that the pseudo-metric well approximates a valid distance measure.
- We demonstrate the first classification results for persistent radar surveillance; prior art has been restricted to narrow apertures. Further, we give the first published results for classification of civilian sedans, which present very small and

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0031-3203/$ - see front matter
doi:10.1016/j.patcog.2010.05.033

very similar radar signatures, in comparison to military vehicles. To illustrate the generic applicability of attributed point patterns, we also present a small example for latent fingerprints.
- For applications with multiple training samples, we describe multidimensional scaling (MDS) and Landmark MDS (LMDS) as tools for visualization, analysis, and alternative classifiers.

Advances in processing and high performance computing have made it possible to tractably solve optimizations necessary for the registration and classification in a multiclass problem [15,16]. Fig. 1 describes the proposed classification algorithm. Given a database of training images, we extract a set of transformation-variant feature vectors (attributed points) for each image; the database is represented as a set of sets $H=\{H_1, H_2,\ldots,H_i,\ldots\}$. Then, given a query image, we extract a set Q of attributed points and calculate a set distance d_i from the query to each set in the database. Using a nearest neighbor test, the classification C is determined from the minimum set distance between Q and each H_i. Thus,

$$C=\arg\min_i d(Q,H_i). \quad (1)$$

Fig. 1 shows several application specific parameters used in the set distance calculation. The distances between attributed point patterns are minimized under a transformation T. The measurement error covariance matrix Σ is used to calculate a Mahalanobis distance between individual feature vectors. Based on the estimated level of clutter in a query, the parameter K is set to facilitate the best subset match. Finally, the particle swarm optimization (PSO) is run for a specified number of particles and iterations as determined by training.

Notice that green boxes in Fig. 1 are performed offline, while dashed boxes are part of the optional MDS/LMDS chain. If multiple training samples are available, it is possible to generate a matrix of distances d_{ij} between patterns in the union of classes. By applying multidimensional scaling (MDS), the samples are represented in a Euclidean space, $X_i\in\mathbb{R}^n$, for a visualization of class separation and an analysis of the pseudo-metric. Given the points in the Euclidean space, it is possible to train classifiers other than NN, such as a support vector machine (SVM) or a linear discriminant analysis (LDA) classifier. When a measured query sample Q is available, we can map the sample into the Euclidean space using a landmark MDS algorithm (LMDS) [17] for visualization or classification.

The remaining sections are organized as follows. Section 2 describes Fig. 1 with subsections detailing the set distance, MDS analysis/training, and LMDS visualization/classification. In Section 3 we use the proposed approach in two applications: latent fingerprint classification and 10-class vehicle classification using circular synthetic aperture radar (SAR). Section 4 provides a summary and discussion of results.

2. Set classification

2.1. Sets distances using the Hausdorff distance

The Hausdorff distance (HD) is a well-known method for representing the distance between two point sets without having a prior correspondence between the two sets. Huttenlocher et al. [10] applied the classical Hausdorff measure [18] concept to matching point sets. Essentially, the HD is the distance of the most isolated point between Q and H_i. However, an outlier or occlusion could skew an otherwise close registration, in which case, the partial Hausdorff distance (PHD) [10], which is the K th minimum distance between points in the sets, may be used. We apply a more robust form of the PHD called the least trimmed square Hausdorff distance (LTS-HD), which takes the mean of the K minimum distances between point sets [19]. To formally describe the LTS-HD, first let $d_{H_i}(\boldsymbol{q})$ be the distance from any $\boldsymbol{q}\in Q$ to its nearest neighbor in H_i, and let $d_{H_i}(\boldsymbol{q})_{(k)}$ denote the kth value from the sorted sequence of nearest neighbor distances calculated for all members of Q to H_i. The directed LTS-HD may be written

$$h_K(Q,\ H_i)=\frac{1}{K}\sum_{k=1}^{K} d_{H_i}(\boldsymbol{q})_{(k)}. \quad (2)$$

The point sets Q and H_i are not necessarily registered prior to calculating the LTS-HD. Rucklidge [20] investigated minimizing the PHD over rigid transformations with scaling. We build upon this concept by using the LTS-HD and generalizing the underlying norm with the Mahalanobis distance [21]. The resulting set distance is defined by

$$d_i=\min_T h_K(T(Q),H_i), \quad (3)$$

where T is the set transformation. In our application, T defines rigid transformations in a 2D plane; however, T is flexible to fit the desired application such as scaling, shifts in time, or 3D transformations.

The underlying nearest neighbor distances in (2) are calculated from the Mahalanobis distance, here represented as a norm between $\boldsymbol{q}\in Q$ and $\boldsymbol{h}\in H_i$ such that

$$\|\boldsymbol{q}-\boldsymbol{h}\|=\sqrt{(\boldsymbol{q}-\boldsymbol{h})^T\Sigma^{-1}(\boldsymbol{q}-\boldsymbol{h})}, \quad (4)$$

where Σ is the measurement error covariance matrix for the vectors in a point set. The Mahalanobis distance facilitates the comparison of vectors, where the various dimensions have different scales, different error sources, or correlated errors. For example, in radar, some attributes may contain spatial locations of bright reflectors, while other attributes contain information about intensity, polarization, or direction of illumination. Use of an appropriate error covariance matrix increases class separability. In practice, it is typical to estimate the measurement error variances of each feature in the feature vectors to populate the diagonal of Σ; however, determining the off-diagonal covariance terms may improve results.

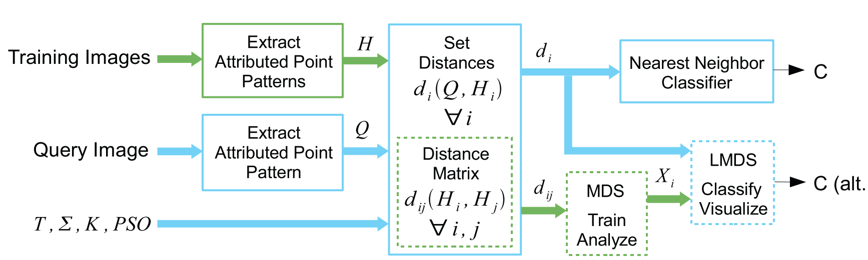

Fig. 1. Calculate the set distance between the query Q and each class sample H_i; the nearest neighbor classification is the shortest distance. An optional chain of processing for analysis/visualization or alternative classifiers using MDS/LMDS is indicated with the dashed boxes. Offline processing is shown in green. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)