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Real-time accurate circle fitting with occlusions

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Abstract

Accurate location of circles inside images is a common problem in many scientific fields. Traditional algorithms, based on fitting a parameterized model, cannot accurately determine the circle in presence of partial occlusions. A novel problem formulation, based on maximum likelihood, allows estimating circles in real-time with sub-pixel accuracy also when occlusions are present.

Keywords: Circle fitting; Maximum-likelihood; Image occlusion; Non-linear optimization

1. Introduction

Accurately locating circles in an image is a challenge in many industrial fields, for instance, video inspection [1], particle tracking [2], robotics [3], neurosurgery [4], archeology [5], biology [6], motion capture [7], and so forth. All these approaches are based on the assumption that the pixels on the circle border or the inner circle pixels have been extracted by suitable algorithms. This task, up to now, cannot be carried out in real-time, in cluttered scenes where the circular object is partially occluded to the view of the surveying camera.

The simplest solution to circle fitting is to compute the barycentre of the cluster of circle pixels (*cluster barycentre* method, *CB*). In this simple approach, the circle radius can be estimated as the maximum distance of the cluster pixels from the estimated centre. Alternatively, only the boundary pixels can be considered (*edge barycentre* method, *EB*); in this case, the radius is computed as the mean distance of the border pixels from the estimated centre. Although simple, these approaches have proved sufficiently accurate in general situations and they have been widely implemented in commercial marker-based motion capture systems [4,7,8]. However, these approaches do not take full advantage of our knowledge of a circle's shape, and more refined parametric approaches have been proposed.

In image processing, the *circular Hough transform* (*CHT*) is widely adopted [9,10]. This technique is based on the fact that a circle can be defined by a triplet of values ($\mathbf{p_C}$ R_C), where $\mathbf{p_C} = (x_C\, y_C)$ is the circle's centre, and R_C its radius. The ($\mathbf{p_C}$ R_C) space is discretized into a finite number of accumulation cells, each corresponding to a specific circle. A counter is associated with each cell. The method requires first extracting all the edge pixels from the image: $\{\mathbf{p_i}\}_{i=1\ldots N}$. Then, for each $\mathbf{p_i}$, the counters of all those cells ($\mathbf{p_C}$ R_C) that are compatible with $\mathbf{p_i}$ are increased by one. After all the points have been considered, the accumulation cell with the highest count contains the most probable circle centre and radius. Despite its easy implementation, the *CHT* method is relatively slow; moreover, it requires a large amount of memory to achieve relatively high accuracy, since accuracy is proportional to the size of the discretized cells [9]. To overcome these problems, efficient implementations of the *CHT* have been proposed, [10]. In particular, methods based on the *randomized Hough transform* (*RHT*) [11,12] have been proposed to speed-up the computation; *CHT* maps a point into a cone in the parameters space, whereas *RHT* iteratively selects a random triplet of points, which is mapped into a single point inside the parameter space. However, *RHT* (and the methods derived from it) does require manual setting of few parameters and it does not reliably identify the circle parameters when the circle is partially occluded and many edge points do not lie on the circumference (see the Discussion section for details).

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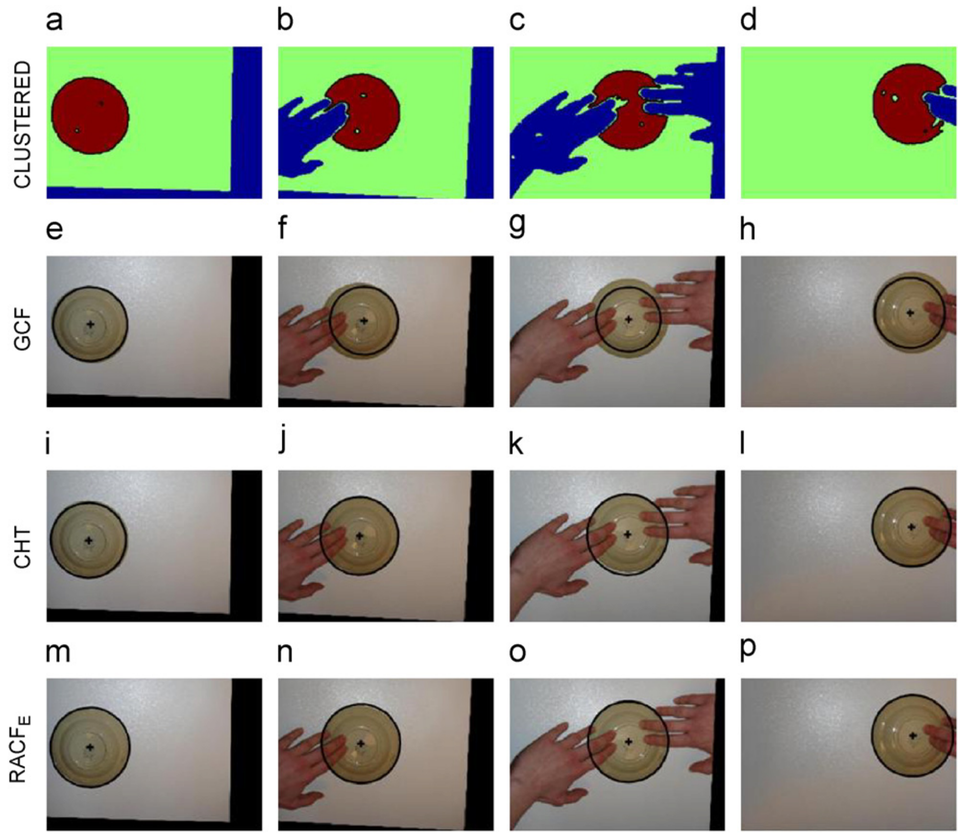

Fig. 1. Four frames extracted from a video of two subjects exchanging a plate. In panels a–d (first row) the original, clustered images are shown; the plate's boundary points are depicted in black. The plate's boundary points are fitted through *GCF* in panels e–h (second row), through *CHT* in panels i–l (third row) and through *RACF* in panels m–p (fourth row—edge-based version, $RACF_E$, is considered here). *GCF* clearly underestimates the circle's radius, while both *CHT* and *RACF* are able to locate the plate accurately also in presence of occlusions. *CHT* requires 13 s on Matlab interpreted language, whereas *RACF* required 80 ms in Matlab and 1.7 ms in C.

An alternative is to consider circle fitting a statistical problem, where the circle's parameters are fit to a set of samples taken on the circle (non-linear regression). In this case, it is again assumed that the a set of N circle points on the circumference, $\{\mathbf{p_i}\}_{i=1\ldots N}$, have been previously extracted by an adequate edge detection algorithm.

In the statistical *algebraic circle fitting* (*ACF*) method [13–15], the parameters vector $\boldsymbol{\theta} = (a\ \mathbf{b}^\mathbf{T}\ c)$, which describes the circumference in analytical form:

$$F_{\boldsymbol{\theta}}(\mathbf{p_i}) = a\mathbf{p_i^T}\mathbf{p_i} + \mathbf{b^T}\mathbf{p_i} + c = 0 \tag{1}$$

is estimated from the set $\{\mathbf{p_i}\}_{i=1\ldots N}$. To avoid ambiguities, the parameter vector $\boldsymbol{\theta}$ is normalized, for example fixing $a = 1$. $\boldsymbol{\theta}$ is computed by minimizing the following cost function:

$$\sum_{i=1}^{N} [F_{\boldsymbol{\theta}}(\mathbf{p_i})]^2, \tag{2}$$

that results in a linear system in $\boldsymbol{\theta}$, from which the circle's centre and its radius are derived. This solution, although simple and fast, does not estimate directly the circle parameters and it is therefore potentially inaccurate [16].

A more principled approach is represented by the statistical *geometrical circle fitting* (*GCF*) method in which the error is defined for each point $\mathbf{p_i}$ as the distance between $\mathbf{p_i}$ and the circumference. The cost function is defined as [16–19]:

$$E = \sum_{i=1}^{N} (\rho_i - R_C)^2, \tag{3}$$

where

$$\rho_i = \|p_i - p_C\| = \sqrt{(x_i - x_C)^2 + (y_i - y_C)^2} \tag{4}$$

is the distance of a generic point from the circle's centre. Under the hypothesis that each measured $\mathbf{p_i}$ is corrupted by Gaussian, isotropic noise, the least squares minimization of Eq. (3) corresponds to the maximum likelihood formulation of the circle fitting problem. The minimization of Eq. (3) is a non-linear problem that has no closed-form analytical solution, and iterative methods must be implemented. Another cost function has been suggested in [17]

$$E = \sum_{i=1}^{N} (\rho_i^2 - R_C^2)^2, \tag{5}$$

which has simpler derivatives for the minimization algorithm, but assigns a heavier weight to outliers [16,18]. In both cases,