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An easy measure of compactness for 2D and 3D shapes

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Abstract

An easy measure of compactness for 2D (two dimensional) and 3D (three dimensional) shapes composed of pixels and voxels, respectively, is presented. The work proposed here is based on the two previous works of the *measure of discrete compactness* [E. Bribiesca, Measuring 2-D shape compactness using the contact perimeter, Comput. Math. Appl. 33 (1997) 1–9; E. Bribiesca, A measure of compactness for 3D shapes, Comput. Math. Appl. 40 (2000) 1275–1284]. The measure of compactness proposed here improves and simplifies the previous measure of discrete compactness. Now, using this proposed measure of compactness, it is possible to compute measures for any kind of object including porous and fragmented objects. Also, the computation of the measures is very simple by means of the use of only one equation. The measure of compactness proposed here depends in large part on the sum of the contact perimeters of the side-connected pixels for 2D shapes or on the sum of the contact surface areas of the face-connected voxels for 3D shapes. Relations between the perimeter and the contact perimeter for 2D shapes and between the area of the surface enclosing the volume and the contact surface area, are presented.

The measure presented here of compactness is invariant under translation, rotation, and scaling. In this work, the term of compactness does not refer to point-set topology, but is related to intrinsic properties of objects. Finally, in order to prove our measure of compactness, we calculate the measures of discrete compactness of different objects. Also, we present an important application for brain structures quantification by means of the use of the new proposed measure of discrete compactness.

Keywords: Measure of compactness; Discrete compactness; Contact perimeter; Contact surface area; Shape analysis; Shape classification; Fragmented objects; Porous objects; Brain images

1. Introduction

The *compactness C* of an object is a beautiful property. The compactness for a 2D shape relates its *perimeter* with its *area* and can be measured by the ratio $(\text{perimeter}^2)/\text{area}$, which is dimensionless and minimized by a disk [1]. In 3D domain, the compactness of an object relates the *enclosing surface area* with the *volume* and can be defined by the ratio $(\text{area}^3)/(\text{volume}^2)$, which is dimensionless and minimized by a sphere [1].

The classical measures of compactness described by Duda and Hart [2], Ballard and Brown [1], Youssef [3], Levine [4], González and Wintz [5], and Haralick and Shapiro [6], depend in large part on the perimeter in 2D domain or on the enclosing surface area in 3D, which produces a sensitive measure to noise. In real applications, most objects have noisy perimeters or enclosing-surfaces, which affect their measures of compactness.

Other authors have proposed different techniques for measuring circularity and compactness of objects: a measure for circularity of digital figures was proposed by Haralick [7]; the compactness of subsets of digital pictures was presented by Sankar and Krishnamurthy [8]; a new distance mapping and its use for shape measurement on binary patterns was developed by Wahl [9]; circularity measures based on mathematical morphology were defined by Ruberto and Dempster [10].

We present here an improvement and simplification on the measure of discrete compactness [11,12] for 2D and 3D shapes composed of pixels and voxels, respectively, which depends in large part on the sum of the contact perimeters of the side-connected pixels for 2D shapes or on the sum of the contact

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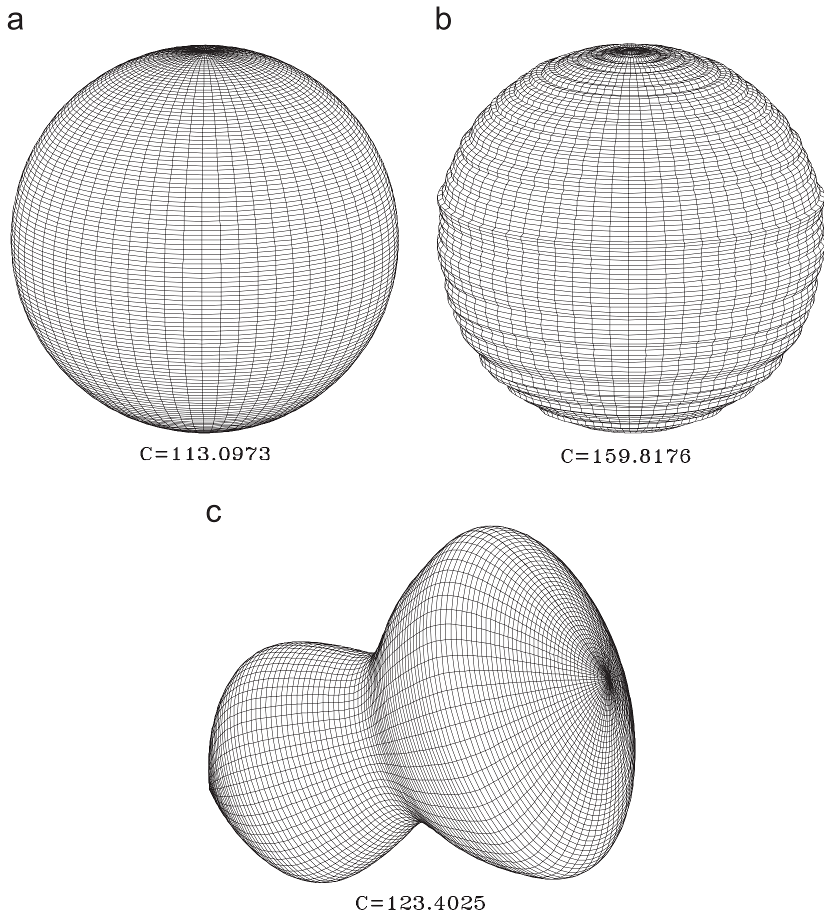

Fig. 1. Examples of solids and their measures of classical compactness: (a) a sphere; (b) the same sphere which is shown in (a) holding a noisy enclosing-surface; and (c) an example of a rigid solid.

surface areas of the face-connected voxels for 3D shapes. Also, we compute measures of discrete compactness for fragmented and porous objects.

Some authors have used the measure of discrete compactness for different and interesting applications. Bogaert et al. [13] presented alternative area-perimeter ratios for measurement of 2D shape compactness of habitats. They state that shape compactness is of main importance to evaluate the effect of external disturbance on natural habitats. In 3D domain, Braumann et al. [14] present the quantification of cervical carcinoma invasion fronts in 3D. For the invasion quantification they refer to discrete compactness which is considered to be in tight correspondence to those invasion features.

In the content of this work, we use volumetric representations for rigid solids by means of spatial occupancy arrays. The solids are represented as 3D arrays of voxels which are marked as filled with matter. Spatial occupancy arrays are very common in computer-aided tomography [1]. We present some examples of measures of compactness for brain structures quantification by means of the use of the new proposed measure of discrete compactness.

The main characteristics and differences between the previous measure of discrete compactness and the new proposed measure of discrete compactness are as follows:

Characteristics of the previous measure of discrete compactness.

1. It measures compactness for connected objects.
2. It is invariant under translation, rotation, and scaling.
3. In order to compute the measure of compactness of any object, first it is necessary to compute the minimum and maximum contact perimeters for 2D shapes or the minimum and maximum contact surfaces for 3D shapes, respectively. Second using these values, measures of compactness are computed.
4. It varies continuously from 0 to 1.

Characteristics of the new measure of compactness proposed here.

1. It measures compactness for connected and disconnected objects (fragmented objects).
2. It is invariant under translation, rotation, and scaling.
3. The computation of the measures of compactness for objects is very simple by means of the use of only one equation. Eq. (8) for 3D shapes and (9) for 2D shapes, respectively
4. It varies continuously from 0 to 1.