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Two symmetrical thinning algorithms for 3D binary images, based on P-simple points

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Abstract

In this paper, we propose two 3D symmetrical thinning algorithms based on the parallel deletion of P-simple points. The first one permits surface skeletons to be obtained. The second one permits curve skeletons to be extracted, and as far as we know, this is the only symmetrical curve thinning algorithm which preserves topology.

These algorithms have been conceived in order to obtain precise results on simple specific objects (parallelepipeds). Consequently, we can predict the number of deletion iterations, the number of deleted points, and the skeleton of these objects obtained by the two algorithms, which is hardly ever possible to assess with other algorithms even on these simple objects.

Keywords: Digital topology; Simple point; Topological number; P-simple point; 3D symmetrical thinning algorithm; Topology preservation

1. Introduction

1.1. Simple points and topological numbers

Some medical or graphical applications require the transformation of objects while preserving their topology. That leads to the well-known notion of a simple point: a point in a binary image is said to be *simple* if its deletion from the image "preserves the topology" [1–15]. One of the authors has proposed the notion of topological numbers of a point [9]: they are the numbers of connected components of the object and of its complement in certain neighborhoods of this point. Certain values of these topological numbers allow to efficiently check whether this point is simple or not [16]: more precisely, a simple point may be locally characterized with topological numbers, (i.e., the examination of only the $3 \times 3 \times 3$ neighborhood centered around a point is enough to decide whether this point is simple or not). More generally, topological numbers have led to a topological classification of points of $\mathbb{Z}^3$ [17], partially used in this paper.

Let us consider Fig. 1 which depicts a three-dimensional object in a cubic grid: the points of the object (resp. the complementary of the object) are represented by black (resp. white) discs, simple black points of the object are encircled (when the so-called 26-adjacency and 6-adjacency are, respectively, used for the object and its complementary, see [5], we also suppose that points outside this figure belong to the complementary of the object).

1.2. Thinning algorithms

The notion of a simple point is fundamental for all transformations where some topological features are to be preserved. Thinning algorithms are usually designed as processes which remove simple points and obey several other criteria. In fact, during the thinning process, certain simple points are kept in order to preserve some geometrical properties of the object.

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Such points are called *end points*. For the 3D case, we can define two different kinds of end points: curve-end points [18–31] and surface-end points [19,32,24,25,27–30,33–36]. A thinning process which preserves curve-end points (resp. surface-end points) is called a *curve thinning algorithm* (resp. a *surface thinning algorithm*). The result obtained by a curve thinning algorithm (resp. a surface thinning algorithm) is called a *curve skeleton* (resp. a *surface skeleton*). See Fig. 2, for an example of a curve skeleton and of a surface skeleton.

In the field of medical applications, thinning is used for information extraction from 3D images and is usually a preliminary step in a sequence of operations on images: for example, thinning permits the reconstruction of 3D vascular trees [37,38]; skeletons may be considered as paths to guide cameras in Computed Tomography Colonography [39,40]; thinning permits the assessment of osteoporosis density studies [41].

1.3. Parallel thinning algorithms

A major problem which arises when designing thinning algorithms is that the simultaneous removal of simple points may change the topology of an object: for example, we see that, if we delete in parallel all simple points of the object depicted in Fig. 1, it will be disconnected.

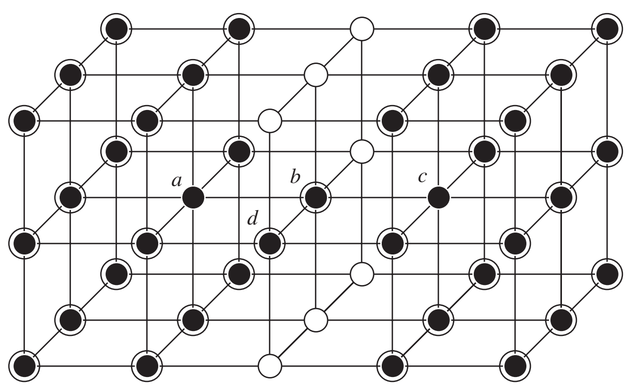

Fig. 1. A three-dimensional object.

To solve this problem, three different solutions may be considered:

- Either deletable points are based on several $3 \times 3 \times 3$ masks, i.e., points which may be deleted must match at least one amongst several given masks or templates. Such a strategy may lead to algorithms based on subiterations, which consists in dividing a deletion iteration into several subiterations. These subiterations may be based on directions [20,32,22,23,26,27] or on subgrids [24–26,29]. Note that the templates are proposed in such a way that the algorithm based on these templates preserves the topology.
- Or it allows to access to an extended neighborhood (i.e., a neighborhood which strictly includes the $3 \times 3 \times 3$ neighborhood centered around a considered point), which conveys more informations. Such a strategy may lead to fully parallel thinning algorithms [21,34] or to symmetrical thinning algorithm [42] (in this paper, we introduce the new notion of a *symmetrical thinning algorithm* to distinguish between parallel algorithms (not based on subiterations) which either use directional conditions (fully parallel) or not (symmetrical), see further details later).
- Or another class of simple point must be found in such a way that if we delete in parallel such points, then the topology is preserved. This is what has been accomplished by the introduction of P-simple points [33]. In fact, this notion is very general and leads to different thinning schemes according to a certain strategy [43]. Examples include directional [44,18,19,33,35,36,31] and symmetrical [30] cases. In Section 1.4, we give an example of thinning scheme based on the parallel deletion of P-simple points.

1.4. P-simple points

One of the authors has proposed the notion of a P-simple point [33]. Let us consider a subset X of $\mathbb{Z}^3$, a subset P of X,

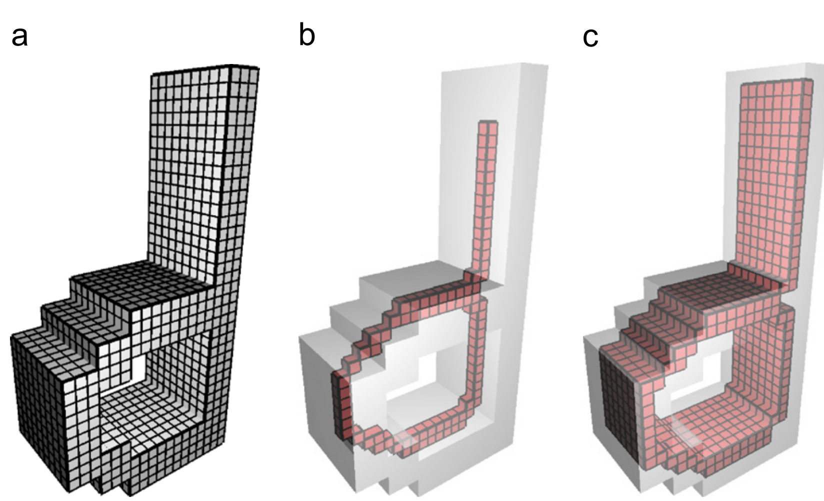

Fig. 2. (a) Initial object, (b) the curve skeleton, (c) and the surface skeleton obtained with our algorithms.