Pattern Recognition 47 (2014) 603–611

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Quaternion Bessel–Fourier moments and their invariant descriptors for object reconstruction and recognition

Zhuhong Shao [a], Huazhong Shu [a,d,*], Jiasong Wu [a,d], Beijing Chen [e], Jean Louis Coatrieux [b,c,d]

[a] Laboratory of Image Science and Technology, School of Computer Science and Engineering, Southeast University, 210096 Nanjing, China
[b] INSERM, U1099, Rennes, F-35000, France
[c] Université de Rennes 1, LTSI, Rennes, F-35042, France
[d] Centre de Recherche en Information Médicale Sino-français (CRIBs), Rennes, F-35042, France
[e] School of Computer & Software, Nanjing University of Information Science & Technology, 210044 Nanjing, China

ARTICLE INFO

Article history:
Received 9 February 2013
Received in revised form
11 July 2013
Accepted 19 August 2013
Available online 28 August 2013

Keywords:
Quaternion Bessel–Fourier moment
Phase
Color image
Invariant descriptor
Recognition

ABSTRACT

In this paper, the quaternion Bessel–Fourier moments are introduced. The significance of phase information in quaternion Bessel–Fourier moments is investigated and an accurate estimation method for rotation angle is described. Furthermore, a new set of invariant descriptors based on the magnitude and the phase information of quaternion Bessel–Fourier moments is derived. Experimental results show that quaternion Bessel–Fourier moments lead to better performance for color image reconstruction than the other quaternion orthogonal moments such as quaternion Zernike moments, quaternion pseudo-Zernike moments and quaternion orthogonal Fourier–Mellin moments. In addition, the angles estimated by the proposed moments are more accurate than those obtained by using other quaternion orthogonal moments. The proposed invariant descriptors show also better robustness to geometric and photometric transformations.

1. Introduction

Moments and moment invariants have been widely used in image processing for pattern recognition [1–3] and image analysis [4–7]. Recently, a new orthogonal moment, the Bessel–Fourier moment, was reported by Xiao [8], wherein the rotation invariance is discussed. However, most of the works reported so far focused on the magnitude coefficients of moments to achieve the rotation invariance and disregarded their corresponding phase information. Unlike the previous works, Li [9] proposed the invariant Zernike moment (ZM) descriptors by combining the ZM magnitude and their phase coefficients as the shape features, which perform better than those described by magnitude-only when applied to noisy image retrieval. Along the same line, another Zernike descriptor preserving the invariance to rotation was described in [10] and used for 2D/3D object recognition. Chen [11] also introduced a ZM phase descriptor, where the ZM magnitude coefficients served as the weighting factors. These works mainly concentrated on gray images or single-channel images.

The quaternion, which can be viewed as the generation of traditional complex number, was introduced by Hamilton in 1843 [12]. The advantage of using the quaternion theory to handle color image is that the existing correlation between color components can be taken into consideration. In the past decades, some quaternion-based techniques have been successfully used for color image processing, for instance the quaternion Fourier transforms [13,14] applied to motion estimation [15] and color image registration [16], the dual-tree quaternion wavelets for multi-scale image processing [17]. A set of invariants with respect to geometric transformations (rotation, scaling and translation), based on the magnitude of quaternion Fourier–Mellin moments, has been originally derived by Guo [18]. More recently, the quaternion Zernike moments as well as the quaternion pseudo-Zernike moments and their corresponding invariants were introduced in [19,20], respectively.

Quaternion Bessel–Fourier moments (QBFMs) and their invariant descriptors are here addressed. This paper is organized as follows. Section 2 first provides some preliminaries about the conventional Bessel–Fourier moments for gray image and the quaternion theory. The quaternion Bessel–Fourier moments for color image and their algorithms are detailed in Section 3. The significance of phase information in the QBFMs is investigated in Section 4. Moreover, a new set of quaternion Bessel–Fourier moment descriptors based on angle estimation is specified. Several experiments are carried out in Section 5 to show

* Corresponding author at: Laboratory of Image Science and Technology, School of Computer Science and Engineering, Southeast University, 210096 Nanjing, China. Tel.: +86 25 83 79 42 49; fax: +86 25 83 79 26 98.
E-mail address: shu.list@seu.edu.cn (H. Shu).

0031-3203/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.patcog.2013.08.016

the performance of quaternion Bessel–Fourier moments and the proposed descriptors by contrast with a family of quaternion orthogonal moments including quaternion Zernike moments, quaternion pseudo-Zernike moments and quaternion orthogonal Fourier–Mellin moments as well as their corresponding invariants.

2. Preliminaries

In this section, we briefly review the conventional orthogonal Bessel–Fourier moments for gray image and some basic properties of the quaternion.

2.1. Bessel–Fourier moments

Considering a gray image $f(\rho,\ \theta)$ defined in polar coordinates, the Bessel–Fourier moment of order n with repetition m is defined as [8]

$$B_{n,m}(f)=\frac{1}{2\pi a_n}\int_0^{2\pi}\int_0^1 J_\nu(\lambda_n\rho)f(\rho,\theta)e^{-jm\theta}\rho d\,\rho d\theta,$$
$$n=0,\ 1,\ 2,\dots,m=0,\ \pm1,\ \pm2,\dots, \tag{1}$$

where the Bessel functions of the first kind are defined as [21]

$$J_\nu(x)=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\Gamma(\nu+k+1)}\left(\frac{x}{2}\right)^{\nu+2k}. \tag{2}$$

Here ν ($\nu\geq0$) is the order of the function and $\Gamma(x)$ the gamma function, λ_n is the n-th zero of the Bessel polynomial and $a_n=[J_{\nu+1}(\lambda_n)]^2/2$ is the normalization constant.

By using Euler's formula, Eq. (1) can be expressed as

$$B_{n,m}(f)=\text{Re}(B_{n,m}(f))+j\text{Im}(B_{n,m}(f)), \tag{3}$$

where Re(x) represents the real part of the complex x, and Im(x) the imaginary part.

2.2. Quaternion

A quaternion q with one real part and three imaginary parts is given by

$$q=a+ib+jc+kd, \tag{4}$$

where a, b, c, d are real numbers and i, j, k are orthogonal imaginary units obeying the following rules

$$i^2=j^2=k^2=-1,\ ij=-ji=k,\ jk=-kj=i,\ ki=-ik=j \tag{5}$$

As shown in (5), the quaternion multiplication is not commutative. If $a=0$, then $q=ib+jc+kd$ is called a pure quaternion.

The conjugate of a quaternion q is $\overline{q}=a-ib-jc-kd$. For any two quaternion numbers p and q, we have $\overline{p\cdot q}=\overline{q}\cdot\overline{p}$. The quaternion q can also be expressed into polar form as [22]: $q=|q|e^{I\theta}$, where $|q|=\sqrt{a^2+b^2+c^2+d^2}$ is called norm, $I=ib+jc+kd/\sqrt{b^2+c^2+d^2}$ and $\theta=\tan^{-1}(\sqrt{b^2+c^2+d^2}/a)$ represent the eigenaxis and the eigenangle, respectively.

3. Quaternion Bessel–Fourier moments

3.1. Definitions

Let $f(\rho,\ \theta)$ be an RGB color image defined in polar coordinates. By taking the red, green and blue channels as three imaginary parts, then a quaternion-based model for color image can be represented as

$$f(\rho,\ \theta)=if_R(\rho,\ \theta)+jf_G(\rho,\ \theta)+kf_B(\rho,\ \theta) \tag{6}$$

Due to the non-commutative property of quaternion multiplication, there are two types of quaternion Bessel–Fourier moments (QBFMs) based on the Bessel function of the first kind. The right-side QBFMs are defined by

$$B^r_{n,m}(f)=\frac{1}{2\pi a_n}\int_0^{2\pi}\int_0^1 J_\nu(\lambda_n\rho)f(\rho,\ \theta)e^{-\mu m\theta}\rho d\rho d\theta,$$
$$n=0,\ 1,\ 2,\dots,m=0,\ \pm1,\ \pm2,\dots \tag{7}$$

Correspondingly, the left-side QBFMs are given by

$$B^l_{n,m}(f)=\frac{1}{2\pi a_n}\int_0^{2\pi}\int_0^1 e^{-\mu m\theta}J_\nu(\lambda_n\rho)f(\rho,\theta)\rho d\rho d\theta, \tag{8}$$

where μ is a unit pure quaternion. In this paper, ν is set to 1 and $\mu=(i+j+k)/\sqrt{3}$.

According to the anti-involution property of quaternion conjugation, the left-side and right-side QBFMs for the same color image have the following relationship:

$$\begin{aligned}B^l_{n,m}(f)&=\overline{\overline{\frac{1}{2\pi a_n}\int_0^{2\pi}\int_0^1 e^{-\mu m\theta}J_\nu(\lambda_n\rho)f(\rho,\theta)\rho d\rho d\theta}}\\&=-\overline{\frac{1}{2\pi a_n}\int_0^{2\pi}\int_0^1 J_\nu(\lambda_n\rho)f(\rho,\theta)e^{\mu m\theta}\rho d\rho d\theta}\\&=-\overline{B^r_{n,-m}(f)}.\end{aligned} \tag{9}$$

In the following sections, the QBFMs refer to the right-side type.

It can be easily verified that the inverse transform of (7) is given by

$$f(\rho,\ \theta)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}J_\nu(\lambda_n\rho)B^r_{n,m}(f)e^{\mu m\theta}. \tag{10}$$

When only a finite number of the QBFMs with order up to P is used, (10) is approximated by

$$\tilde{f}(\rho,\theta)=\sum_{n=0}^{P}\sum_{m=-n}^{n}J_\nu(\lambda_n\rho)B^r_{n,m}(f)e^{\mu m\theta}. \tag{11}$$

For digital images, the double integral in Eq. (7) is substituted by a double summation. Its discrete form is given by

$$B^r_{n,m}(f)=\frac{1}{2\pi a_n(N-1)^2}\sum_{x=1}^{N}\sum_{y=1}^{N}J_\nu(\lambda_n\rho)f(x,y)e^{-\mu m\theta}, \tag{12}$$

where N is the number of pixels in each coordinate axis of the image, the parameters ρ and θ are computed by the mapping transformation as follows [7]

$$\rho=\sqrt{(c_1x+c_2)^2+(c_1y+c_2)^2},\ \theta=\tan^{-1}\left(\frac{c_1y+c_2}{c_1x+c_2}\right),$$
$$c_1=\frac{\sqrt{2}}{N-1},\ c_2=-\frac{1}{\sqrt{2}}.$$

3.2. Algorithm

Substituting (6) into (7) and combining (3), we have

$$\begin{aligned}B^r_{n,m}(f)&=\frac{1}{2\pi a_n}\int_0^{2\pi}\int_0^1 J_\nu(\lambda_n\rho)f(\rho,\theta)e^{-\mu m\theta}\rho d\rho d\theta\\&=\frac{1}{2\pi a_n}\int_0^{2\pi}\int_0^1 J_\nu(\lambda_n\rho)(if_R(\rho,\theta)+jf_G(\rho,\theta)\\&\quad+kf_B(\rho,\theta))(\cos(m\theta)-\mu\sin(m\theta))\rho d\rho d\theta\\&=i[\text{Re}(B_{n,m}(f_R))+\mu\text{Im}(B_{n,m}(f_R))]+j[\text{Re}(B_{n,m}(f_G))\\&\quad+\mu\text{Im}(B_{n,m}(f_G))]+k[\text{Re}(B_{n,m}(f_B))+\mu\text{Im}(B_{n,m}(f_B))]\\&=A_{n,m}+iX_{n,m}+jY_{n,m}+kZ_{n,m},\end{aligned} \tag{13}$$

where

$$A_{n,m}=\frac{-1}{\sqrt{3}}[\text{Im}(B_{n,m}(f_R))+\text{Im}(B_{n,m}(f_G))+\text{Im}(B_{n,m}(f_B))],$$