Pattern Recognition 47 (2014) 623–633

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Unsupervised segmentation and approximation of digital curves with rate-distortion curve modeling

Alexander Kolesnikov [a,*], Tuomo Kauranne [b]

[a] Arbonaut Ltd., Kaislakatu 2, FI-80130 Joensuu, Finland
[b] Lappeenranta University of Technology, PO. Box 20, FI-53851 Lappeenranta, Finland

ARTICLE INFO

Article history:
Received 31 December 2012
Received in revised form
4 August 2013
Accepted 8 September 2013
Available online 16 September 2013

Keywords:
Shape
Graphical model
Piecewise linear approximation
Curve fitting

ABSTRACT

This paper considers the problem of unsupervised segmentation and approximation of digital curves and trajectories with a set of geometrical primitives (model functions). An algorithm is proposed based on a parameterized model of the Rate–Distortion curve. The multiplicative cost function is then derived from the model. By analyzing the minimum of the cost function, a solution is defined that produces the best possible balance between the number of segments and the approximation error. The proposed algorithm was tested for polygonal approximation and multi-model approximation (circular arcs and line segments for digital curves, and polynomials for trajectory). The algorithm demonstrated its efficiency in comparisons with known methods with a heuristic cost function. The proposed method can additionally be used for segmentation and approximation of signals and time series.

1. Introduction

Polygonal approximation of digital curves is used in image processing, computer graphics, pattern recognition, data retrieval, CAD, GIS, shape analysis and encoding. In all above applications, it is used to reduce the amount of data that need to be processed, stored and transmitted. In addition to polygonal approximation, signals, curves and trajectories can be approximated with non-linear functions like arcs, polynomials and splines [1–7]. There are two main approaches [8] to the problem of shape representation with a polygonal curve: (1) identification of the dominant points as vertices of the approximating polygon, or (2) approximation of the segments of the input curve with a continuous sequence of line segments. The former approach is based on analysis of the curvature or other features to identify the corner points of the curve [9–13]. The approach has the drawback that the algorithms are sensitive to contour noise and often include input or hidden parameters. When using the latter approach, the problem of the approximation of digital curves is usually formulated in one of the following two forms [7,14]:

(1) *Min-# problem*: With a given constraint on the approximation error, approximate a curve with the minimum number of line segments (the description length).

(2) *Min-ε problem*: With a given number of approximating line segments M (description length R), approximate the input curve with a minimum approximation error.

The solution to the problem depends on the error measure in use. For an error measure of L_∞ (the maximum deviation) the solution can be found with the following heuristic [15–20] or optimal algorithms [21–24]. With L_2-norm (Integral Square Error, ISE), the problem can be solved with heuristic [20,25] or optimal algorithms [26–31].

Thus, an approximation can be found if the number of segments M (the description length R) or the error bound ε_0 is known. However, before an approximation can be formed, we need to know the most suitable value of the input parameter and how many segments are required to represent the curve adequately. Therefore, a method is needed that determines the input parameter of the approximation algorithm for a concrete curve.

A possible approach to the problem is to find a balance between the number of segments M and the approximation error E by introducing a cost function that incorporates both M and E. For an additive error measure L_2, the Lagrange multiplier method can be used. The Lagrange multiplier algorithm searches for a solution with a minimal value of the *additive* cost function $C=E_2+\lambda M$ [2,4,5,32], where E_2 is the Integral Square Error. The trade-off between the number of segments M and the Integral Square Error E_2 is controlled by means of the user-defined heuristic Lagrange multiplier λ.

In [9], a *multiplicative* criterion *Figure of Merit* (*FOM*) was introduced for evaluating solutions that have been obtained by using heuristic algorithms for polygonal approximation: $FOM=E_2M$.

* Corresponding author. Tel.: +358 468 104 853.
E-mail addresses: alexander.kolesnikov@arbonaut.com (A. Kolesnikov), tuomo.kauranne@lut.fi (T. Kauranne).

0031-3203/$ - see front matter
http://dx.doi.org/10.1016/j.patcog.2013.09.002

Later, this criterion was presented as an "optimization error" [10], "weighted sum of square error" [11] and "compromise ratio" [13]. However, Rosin [33] has demonstrated that the two terms, M and E_2, in the *FOM* are not balanced – the heuristic criterion gives solutions with too many linear segments. In the modified criterion *FOM-n* ($n=2$ or 3) proposed in [12], the number of segments M was penalized by raising it to a power n that would have the effect of reducing this bias: $FOM-n=E_2M^n$.

The criterion *FOM*-2 was proposed earlier [17], as the "relative error", in the following form: $E_r=M\sqrt{E_2}$, but this publication remained unnoticed. Although originally created as a criterion for evaluating heuristic algorithms [8,12,17,25], the criterion *FOM-n* has been used for determining the number of linear segments [20,34–36]. However, it is still not clear which value of the parameter n is more appropriate, $n=2$ or $n=3$.

For approximation with the error measure L_∞, the multiplicative criterion "*weighted maximum error*", WE_∞ was introduced in [11]: $WE_\infty=E_\infty M$. The *modified Figure of Merit* (*MFOM*-3) was proposed in [19]. The criterion incorporates the criterion *FOM*-2 and the maximum deviation E_∞ with the number of segments M as follows:

$$MFOM\text{-}2=MFOM\text{-}2E_\infty M=E_2E_\infty M^3 \tag{1}$$

In the algorithm [19], the set of polygonal approximating curves was sequentially constructed for input contour with a heuristic algorithm for increasing error tolerance for maximum deviation until the first local minimum of the *MFOM*-3 criterion is reached.

In this paper, we approach the problem from another angle. Based on the fractal properties of curves, we first construct a parameterized model of the approximation error as a function of the number of segments M. Then, we derive a multiplicative cost function for approximation with error measures L_2 and L_∞. The minimum of the cost function gives us the optimal number and type of approximating segments.

The paper is organized as follows. Section 2 discusses the problem of segmentation and approximation of curves. In Section 3, we introduce a parameterized model of the Rate–Distortion. Section 4 presents a criterion that gives a solution to the problem in question. Section 5 sets out the results of our experiments for test curves. Section 6 presents the conclusions.

2. Rate–Distortion curve

An open planar N-vertex curve P is defined as ordered set of points: $P=\{p(1), \ldots, p(N)\}$, where $p(n)=(x(n), y(n))$. The polygonal curve P is approximated by another polygonal curve $Q=\{q(1), \ldots, q(M+1)\}$ with M linear segments, where $q(m)=p(i_m)$.

The approximation error with the measure L_∞ for a curve segment $\{p(i_m), \ldots, p(i_{m+1})\}$ is defined as the maximum of the Hausdorff distance d_H between points $p(n)$ of the curve segment and the corresponding approximating linear segment $S(i_m, i_{m+1})$ defined by the points $p(i_m)$ and $p(i_{m+1})$. Maximum deviation for the curve P is defined as follows:

$$\delta(M)\equiv E_\infty(M)=\max_{1\le m\le M}\left\{\max_{i_m\le n\le i_{m+1}}\{d_H(p(n),S(i_m,i_{m+1}))\}\right\}. \tag{2}$$

The approximation error with the measure L_2 for a curve segment $\{p(i_m), \ldots, p(i_{m+1})\}$ is defined as the sum of squared

```
DP_PA_L∞ (P: array, M: integer)
// P={p(1), ..., p(N)}: input curve;
// M2: The upper bound of the segments number;
Dist(i,j) ← ∞, i=[1, N], j=[1,M2]
Dist(1,0) ← 0
Err(1,0) ← 0
FOR n=2 TO N DO
  // Error for curve segments {pj, …, pn}
  FOR j=1 TO n-1 DO
     e2(n-j) ← E2(j,n)
     d(n-j) ← E∞(j,n)
  END
  // Search of the error minimum
  FOR m=1 TO M2 DO
    FOR j=m TO n-1 DO
      C ←max{Dist(j,m-1), d(n-j)}
      IF(Dist(n,m) = C)
        IF(Err(j,m)+e2(n-j)<Err(n,m)
           Err(n,m) ← Err(j,m)+e2(n-j)
           A(n,m) ← j
        ENDIF
      ENDIF
      IF(Dist(n,m) < C)
         Dist(n,m) ← C
         A(n,m) ← j
         Err(n,m) ← Err(j,m)+e2(n-j)
      ENDIF
    ENDFOR j
  ENDFOR m
ENDFOR n
```

```
DP_PA_L2(P: array, M2: integer)
  // P={p(1), ..., p(N)}: input curve;
  // M2: The upper bound of the segments number;
  Err(1,0) ← 0;
  FOR n=2 TO N DO
    // Error for curve segments {pj, …, pn}
    FOR j=1 TO n-1 DO
      e2(n-j)←E2(j,n);
    END
    // Search of the error minimum
    FOR m=1 TO M2 DO
      FOR j=m TO n-1 DO
        IF(Err(j,m)+e2(j-n)<Err(n,m)
          Err(n,m) ← Err(j,m)+e2(n-j)
          A(n,m) ← j
        ENDIF
      ENDFOR j
    ENDFOR m
  ENDFOR n
```

Fig. 1. General scheme of the full-search DP algorithm for polygonal approximation for error measures L_2 (left) and L_∞ (right) [26]. M.