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A sparse nonnegative matrix factorization technique for graph matching problems

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ABSTRACT

Graph matching problem that incorporates pairwise constraints can be cast as an Integer Quadratic Programming (IQP). Since it is NP-hard, approximate methods are required. In this paper, a new approximate method based on nonnegative matrix factorization with sparse constraints is presented. Firstly, the graph matching is formulated as an optimization problem with nonnegative and sparse constraints, followed by an efficient algorithm to solve this constrained problem. Then, we show the strong relationship between the sparsity of the relaxation solution and its effectiveness for graph matching based on our model. A key benefit of our method is that the solution is sparse and thus can approximately impose the one-to-one mapping constraints in the optimization process naturally. Therefore, our method can approximate the original IQP problem more closely than other approximate methods. Extensive and comparative experimental results on both synthetic and real-world data demonstrate the effectiveness of our graph matching method.

1. Introduction

Many problems in computer vision and pattern recognition can be formulated as a problem of finding consistent correspondences between two sets of features. In computer vision, the problem of establishing correspondences between two feature sets can be effectively solved by Attributed Relational Graph (ARG) matching. In general, an ARG consists of nodes with unary attributes and edges with binary relationships between the nodes. The goal of ARG matching (graph matching) is to find a mapping between two node sets that preserves both unary attributes and binary relationships between nodes as much as possible [1–3]. Previous approaches [4–8] have formulated graph matching as an Integer Quadratic Programming (IQP) problem. Since IQP is known to be NP-hard, graph matching is solved either exactly in a very restricted setting or approximately. Most of the recent literatures on graph matching have followed the second way, i.e., developing approximate relaxations to the graph matching problem [5,6,8,11–13]. Van Wyk et al. [8] proposed a graph matching method by iteratively projecting the approximate correspondence matrix onto a convex domain of desired integer constraints. Leordeanu and Hebert [5] proposed a Simple and efficient approximate Method (SM) to IQP using a spectral relaxation technique. This method can find the global maximum solution of the relaxed problem efficiently by computing the leading eigenvector of the symmetric nonnegative affinity matrix. However, SM ignores the mapping constraints in the relaxation step, and it obtains the final matching solution based on a post-optimization step using a discretization technique. Cour et al. [12] extended SM to Spectral Matching with Affine Constraints (SMAC) by incorporating the affine constraints into the spectral relaxation process. Comparing with SM, it further encodes the one-to-one matching constraints, therefore, it can approximate the original IQP problem more closely. Torresani et al. [7] represented graph matching as an energy minimization problem which can be efficiently optimized by dual decomposition. Leordeanu and Hebert [13] proposed an iterative matching method (IPFP) which optimized the IQP in a discrete domain and therefore can satisfy the one-to-one mapping constraints strictly in the optimization process. It integrates the discretization step and objective function optimization simultaneously, and shows strong climbing and convergence properties. Zhou et al. [27] proposed a matching algorithm by exploiting the properties of factorized affinity matrix. In addition to optimization-based methods, probabilistic frameworks can also be used for solving graph matching problems [9,10,14,15]. Zass and Shashua [9] introduced a probabilistic model for soft hypergraph matching between two complex feature sets. Cho and Lee [10] interpreted graph matching problem based on a random walk model and provided a robust matching algorithm by simulating random walks with re-weighting jumps which enforce the mapping constraints on the associated graph. Caetano et al. [14,15]

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formulated graph matching as a problem of finding a maximum probability configuration in a graphical model. In our work, we focus on optimization-based graph matching methods.

The aim of this paper is to propose a new relaxation method for graph matching problem based on sparse model and Nonnegative Matrix Factorization (NMF) method, which have been widely used in pattern recognition and computer vision community [16–20]. Our main contributions are three aspects. Firstly, a new relaxation problem for graph matching based on NMF with sparse constraints is proposed, followed by an efficient algorithm to solve this relaxation problem. Secondly, the strong relationship between the sparse property of the relaxation solution and its effectiveness for graph matching problem is experimentally demonstrated based on the proposed matching model. Thirdly, we show experimentally that the proposed matching method can generate sparse solution and thus can approximately incorporate the one-to-one mapping constraints in the optimization process. Therefore it achieves a close approximation for the original IQP graph matching problem.

The remainder of this paper is organized as follows: In Section 2, the formulation of graph matching as an IQP problem is introduced. In Section 3, a new relaxation method for graph matching is proposed based on NMF with sparse constraints. In Section 4, some advantages of the proposed sparse NMF matching method are demonstrated. In Section 5, our matching method is applied to some matching tasks on both synthetic graph data and real-world image data.

2. Problem formulation

Assume that two ARGs to be matched are $G^{\mathrm{D}}=(V^{\mathrm{D}}, E^{\mathrm{D}}, A^{\mathrm{D}}, R^{\mathrm{D}})$ and $G^{\mathrm{M}}=(V^{\mathrm{M}}, E^{\mathrm{M}}, A^{\mathrm{M}}, R^{\mathrm{M}})$ where V represents a set of nodes, E, edges, and A unary attributes set, R binary relationships set. Each $v_i^{\mathrm{D}} \in V^{\mathrm{D}}$ or edge $e_{ij}^{\mathrm{D}} \in E^{\mathrm{D}}$ has an associated attribute vector $\mathbf{a}_i^{\mathrm{D}} \in A^{\mathrm{D}}$ or $\mathbf{r}_{ij}^{\mathrm{D}} \in R^{\mathrm{D}}$ [1,10]. The objective of graph matching is to determine the correct correspondences between G^{D} and G^{M}. A correspondence mapping is a set C of pairs (or assignments) $(v_i^{\mathrm{D}}, v_{i'}^{\mathrm{M}})$, where $v_i^{\mathrm{D}} \in V^{\mathrm{D}}$ and $v_{i'}^{\mathrm{M}} \in V^{\mathrm{M}}$. For each assignment $c_i=(v_i^{\mathrm{D}}, v_{i'}^{\mathrm{M}})$ in C, there is an affinity $\mathbf{W}_{c_i,c_i}=f_a(\mathbf{a}_i^{\mathrm{D}}, \mathbf{a}_{i'}^{\mathrm{M}})$ that measures how well the node $v_i^{\mathrm{D}} \in V^{\mathrm{D}}$ matches the node $v_{i'}^{\mathrm{M}} \in V^{\mathrm{M}}$. Also, for each pair of assignments (c_i, c_j), where $c_i=(v_i^{\mathrm{D}}, v_{i'}^{\mathrm{M}})$ and $c_j=(v_j^{\mathrm{D}}, v_{j'}^{\mathrm{M}})$, there is an affinity $\mathbf{W}_{c_i,c_j}=f_r(\mathbf{r}_{ij}^{\mathrm{D}}, \mathbf{r}_{i'j'}^{\mathrm{M}})$ that measures how compatible the nodes $(v_i^{\mathrm{D}}, v_j^{\mathrm{D}})$ in the data graph G^{D} are with the nodes $(v_{i'}^{\mathrm{M}}, v_{j'}^{\mathrm{M}})$ in the model graph G^{M}. Therefore, we can use a matrix $\mathbf{W}$ with the diagonal term $\mathbf{W}_{c_i,c_i}$ representing a unary affinity of a correspondence $c_i=(v_i^{\mathrm{D}}, v_{i'}^{\mathrm{M}})$, and the non-diagonal element $\mathbf{W}_{c_i,c_j}$ containing a pairwise affinity between two assignments $c_i=(v_i^{\mathrm{D}}, v_{i'}^{\mathrm{M}})$ and $c_j=(v_j^{\mathrm{D}}, v_{j'}^{\mathrm{M}})$.

The correspondences can be represented by a permutation matrix $\mathbf{X}$ such as $\mathbf{X}_{i,i'}=1$ implies that node v_i^{D} in the data graph G^{D} corresponds to the node $v_{i'}^{\mathrm{M}}$ in model graph G^{M}, and $\mathbf{X}_{i,i'}=0$ otherwise. In this paper, we denote $\mathbf{x} \in \{0,1\}^{nm}$ as a row-wise vectorized replica of $\mathbf{X}$, where $n=|V^{\mathrm{D}}|, m=|V^{\mathrm{M}}|$. In the following, $\mathbf{x}$ is called as the vector form of $\mathbf{X}$, and $\mathbf{X}$ is called as the matrix form of $\mathbf{x}$. The graph matching problem can usually be formulated as an Integer Quadratic Programming (IQP) problem [5,12,13], i.e., finding the indicator vector $\mathbf{x}$ that maximizes the following objective function,

$$\max_{\mathbf{x}} \quad \varepsilon_1(\mathbf{x})=\mathbf{x}^{\mathrm{T}}\mathbf{W}\mathbf{x} \quad s.t. \quad \mathbf{x}_i \in \{0,1\}, \mathbf{A}\mathbf{x}=\mathbf{1} \tag{1}$$

The linear constraint $\mathbf{A}\mathbf{x}=\mathbf{1}$ refers to the orthogonality of the permutation matrix $\mathbf{X}$ ($\mathbf{X}^{\mathrm{T}}\mathbf{X}=\mathbf{X}\mathbf{X}^{\mathrm{T}}=\mathrm{I}$), where $\mathbf{X}$ is the matrix form of $\mathbf{x}$, and ensures one-to-one matching between G^{D} and G^{M}.

It is well known that, the above IQP problem is NP-hard and no efficient algorithm exists. Therefore, many methods have been proposed to find an approximate solution of this optimization problem [2,5,11–13]. These methods usually avoid combinatorial search by approximating the objective function or by relaxing the mapping constraints. In general, from the optimization perspective, a practical relaxation method should approximate the original IQP problem as closely as possible. Specifically, it should satisfy the following two desired matching properties [2,12,13].

(1) It should maximize the objective function as far as possible.
(2) It should satisfy the matching (mapping) constraints as closely as possible.

We call these properties as Objective Property and Constraint Property, respectively. Usually, the conventional approximation methods cannot guarantee that the solution satisfies the matching constraints strictly, and they obtain the final correspondence solution based on a post-optimization step by using some discretization techniques. Leordeanu and Hebert [5] proposed a spectral technique (SM) to graph matching problem. This method is effective for optimizing the objective, i.e., it can find the global maximum of the relaxation problem efficiently. However, the method cannot satisfy the mapping constraints closely because of the relaxation, namely $\|\mathbf{x}\|_2=1$. Recently, they proposed an iterative matching method (IPFP) [13] which integrates the discretization step and objective function optimization simultaneously. This method optimizes the IQP problem in a discrete domain and thus can satisfy the mapping constraints strictly. Cour et al. [12] further extended SM to SMAC by incorporating the matching constraints within the relaxation process. Comparing with SM, this method satisfies the mapping constraints more closely and therefore can obtain more effective solution for graph matching problem. In this paper, we propose a new relaxation matching method based on NMF with sparse constraints. The main features for our sparse NMF graph matching method are two aspects. Firstly, our sparse NMF based solution is sparse and thus approximately incorporates the discrete mapping constraints naturally. Secondly, an efficient algorithm can be derived to optimize the objective function of the relaxation problem.

3. Related work on L_p norm constraint

Given an objective function $J(\mathbf{u})$ for the parameter vector $\mathbf{u}=[\mathbf{u}_1, \mathbf{u}_2 \ldots \mathbf{u}_n]$, the general form of the L_p norm constraint problem can be described as [25,26]

$$\max_{\mathbf{u}} \quad J(\mathbf{u}) \quad s.t. \quad ||\mathbf{u}||_p=b, \tag{2}$$

where b is a positive constant. $\|.\|_p$ denotes the L_p norm and it is defined as

$$||\mathbf{u}||_p=\left(\sum_{i=1}^{n}|\mathbf{u}_i|^p\right)^{1/p} \tag{3}$$

When searching for the optimal solution for this problem, the L_p norm constraint ($||\mathbf{u}||_p=b$) generally imposes a geometric structure to the parameter space. The above L_p norm constraint based problems have been widely used in many tasks such as data mining, pattern recognition, signal processing and so on [16,19,25,26]. Ding et al. [16] has proposed an L_p norm constraint based relaxation model for finding the maximal clique of graph by solving the following optimization problem,

$$\max_{\mathbf{u}} \quad J(\mathbf{u})=\mathbf{u}^{\mathrm{T}}\mathbf{A}\mathbf{u} \quad s.t. \quad ||\mathbf{u}||_p=1, \mathbf{u}_i \geq 0, \tag{4}$$

where $\mathbf{A}$ is the adjacency matrix of an unweighted graph, i.e., $\mathbf{A}_{ij} \in \{0,1\}$, $\mathbf{A}_{ii}=0.p \in [1,2]$ is a parameter.