Pattern Recognition 43 (2010) 2620–2629

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Image analysis by Bessel–Fourier moments

Bin Xiao [a,b,*], Jian-Feng Ma [a], Xuan Wang [c]

[a] The Key Laboratory of Computer Networks and Information Security, Ministry of Education, Xidian University, Xi'an 710071, China
[b] School of Physics & Electrical Information Engineering, Ningxia University, Yinchuan 750021, China
[c] School of Physics and Information Technology, Shaanxi Normal University, Xi'an 710062, China

ARTICLE INFO

Article history:
Received 20 October 2009
Received in revised form
15 March 2010
Accepted 20 March 2010

Keywords:
Bessel function
Orthogonal moments
Images reconstruction
Image recognition
Invariant moments

ABSTRACT

In this paper, we proposed a new set of moments based on the Bessel function of the first kind, named Bessel–Fourier moments (BFMs), which are more suitable than orthogonal Fourier–Mellin and Zernike moments for image analysis and rotation invariant pattern recognition. Compared with orthogonal Fourier–Mellin and Zernike polynomials of the same degree, the new orthogonal radial polynomials have more zeros, and these zeros are more evenly distributed. The Bessel–Fourier moments can be thought of as generalized orthogonalized complex moments. Theoretical and experimental results show that the Bessel–Fourier moments perform better than the orthogonal Fourier–Mellin and Zernike moments (OFMMs and ZMs) in terms of image reconstruction capability and invariant recognition accuracy in noise-free, noisy and smooth distortion conditions.

1. Introduction

Moments due to its ability to represent global features have been used in a variety of applications in image analysis, such as visual pattern recognition [1,2], object classification [3], template matching [4], robot sensing techniques [5], image watermarking [6] and edge detection [7]. Among the different types of moments, Cartesian geometric moments and their extensions in the form of complex moments and radial moments have played important roles. However, these regular moments are not orthogonal. Consequently, the reconstruction of image from these moments is quite difficult and computationally expensive. Moreover, it has a certain degree of information redundancy and high sensitivity to noise [8].

Teague [9] suggested the orthogonal moments based on orthogonal polynomials to overcome the problems associated with the regular moments, such as Zernike and Legendre moments. Zernike moments are able to store image information with minimal information redundancy and have the property of being rotation invariant. It has been widely used in character recognition [10] and image watermarking [11]. Revaud et al. [12] proposed an improved Zernike moments for 2D/3D object recognition. The improved algorithm is more robust against noise and geometric deformation. Legendre moments are constructed using the Legendre polynomials [9]. Teh et al. [13] and Mukundan et al. [14] studied the image representation capability, information redundancy, noise sensitivity and computational aspect of Lengdre moments. Yang et al. [15] used Legendre moments for the analysis of grey level images.

Sheng et al. [16] introduced orthogonal Fourier–Mellin moments (OFMMs) based on a new set of radial polynomials and showed that orthogonal Fourier–Mellin moments have better performance than Zernike moments in terms of image reconstruction capability and noise sensitivity. Kan et al. [10] used the orthogonal Fourier–Mellin moments for invariant character recognition. Papakostas et al. [17] proposed an efficient algorithm for the computation of the orthogonal Fourier–Mellin moments.

The discrete orthogonal moments have been proposed recently. Mukundan et al. [18] introduced a set of discrete orthogonal moment functions based on the discrete Tchebichef polynomials, their study shows that the implement of discrete orthogonal moments does not involve any numerical approximations since the basis functions will exactly satisfy the orthogonal property and yield a superior image reconstruction result. Similarly, other classical discrete orthogonal moments such as Krawtchouk moments [19], dual Hahn moments [20] and Racah moments [21] have been proposed more recently. Legendre, Tchebichef, Krawtchouk, dual Hahn and Racah moments all fall into the same class of orthogonal moments defined in the Cartesian coordinate space, where moment invariants particularly rotation invariants are not readily available. However, Zernike and orthogonal Fourier–Mellin moments are defined in polar

* Corresponding author at: The Key Laboratory of Computer Networks and Information Security, Ministry of Education, Xidian University, Xi'an 710071, China.
E-mail address: xiaobinic@gmail.com (B. Xiao).

0031-3203/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.patcog.2010.03.013

coordinate, rotating the image does not change the magnitude of its moments, its rotation invariant.

In this paper, another new set of orthogonal moments defined in polar coordinate is proposed, namely Bessel–Fourier moments (BFMs). It is based on the Bessel function of the first kind. Similar to Zernike and orthogonal Fourier–Mellin moments, it is easy for image reconstruction and rotation invariant recognition. However, it has better performance than Zernike and orthogonal Fourier–Mellin moments in terms of images reconstruction capability and invariant recognition accuracy due to its good orthogonality. It can be used for rotation-invariant object recognition, (i.e., character or butterfly pictures recognition).

This paper is organized as follows. In Section 2, the definitions of Bessel function of the first kind, Bessel–Fourier moments and Bessel–Fourier moments invariants are presented. Section 3 discusses the properties and performance comparison of Bessel–Fourier, orthogonal Fourier–Mellin and Zernike moments. The comparative analysis of the proposed approach with orthogonal Fourier–Mellin and Zernike moments in terms of the image reconstruction capability, recognition accuracy and computational time is provided in Section 4. Section 5 concludes the paper.

2. Bessel–Fourier moments

Bessel–Fourier moments are a set of moments based on the Bessel function of the first kind. In this section the definition of Bessel function of the first kind is provided, and then the Bessel–Fourier moments and Bessel–Fourier moments invariant are introduced.

2.1. Bessel function of the first kind

The definition of the Bessel function of the first kind is as follows [22,23]:

$$J_v(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma(v+k+1)}\left(\frac{x}{2}\right)^{v+2k} = \frac{(x/2)^v}{\Gamma(v+1)}\,{}_0F_1(v+1,-(x/2)^2) \quad (1)$$

where v is a real constant, $\Gamma(a)$ is the gamma function, ${}_0F_1$ is the generalized hypergeometric function. The Bessel function is the solution of the Bessel's equation

$$y'' + \frac{1}{x}y' + \left(1-\frac{v^2}{x^2}\right)y = 0 \quad (2)$$

and has the following recurrence relations [24]:

$$J_{v-1}(x)+J_{v+1}(x) = \frac{2v}{x}J_v(x) \quad (3)$$

2.2. Bessel–Fourier moments

We define the Bessel–Fourier moments using Bessel function of the first kind in the polar coordinate as

$$B_{nm} = \frac{1}{2\pi a_n}\int_0^{2\pi}\int_0^1 f(r,\theta)J_v(\lambda_n r)\exp(-jm\theta)r\,dr\,d\theta \quad (4)$$

where $f(r,\theta)$ is the image and $n=0,1,2,\ldots, m=0,\pm1,\pm2,\ldots$ is the moment order, $a_n=[J_{v+1}(\lambda_n)]^2/2$ is the normalization constant. $J_v(\lambda_n r)$ is the Bessel polynomial in r of degree n, λ_n is the n-th zero of $J_v(r)$. A plot of the polynomial $J_1(\lambda_n r)$ is given in Fig. 1(a), and the set of $J_v(\lambda_n r)$ is orthogonal over the range $0 \le r \le 1$.

$$\int_0^1 rJ_v(\lambda_n r)J_v(\lambda_k r)\,dr = a_n\delta_{nk} \quad (5)$$

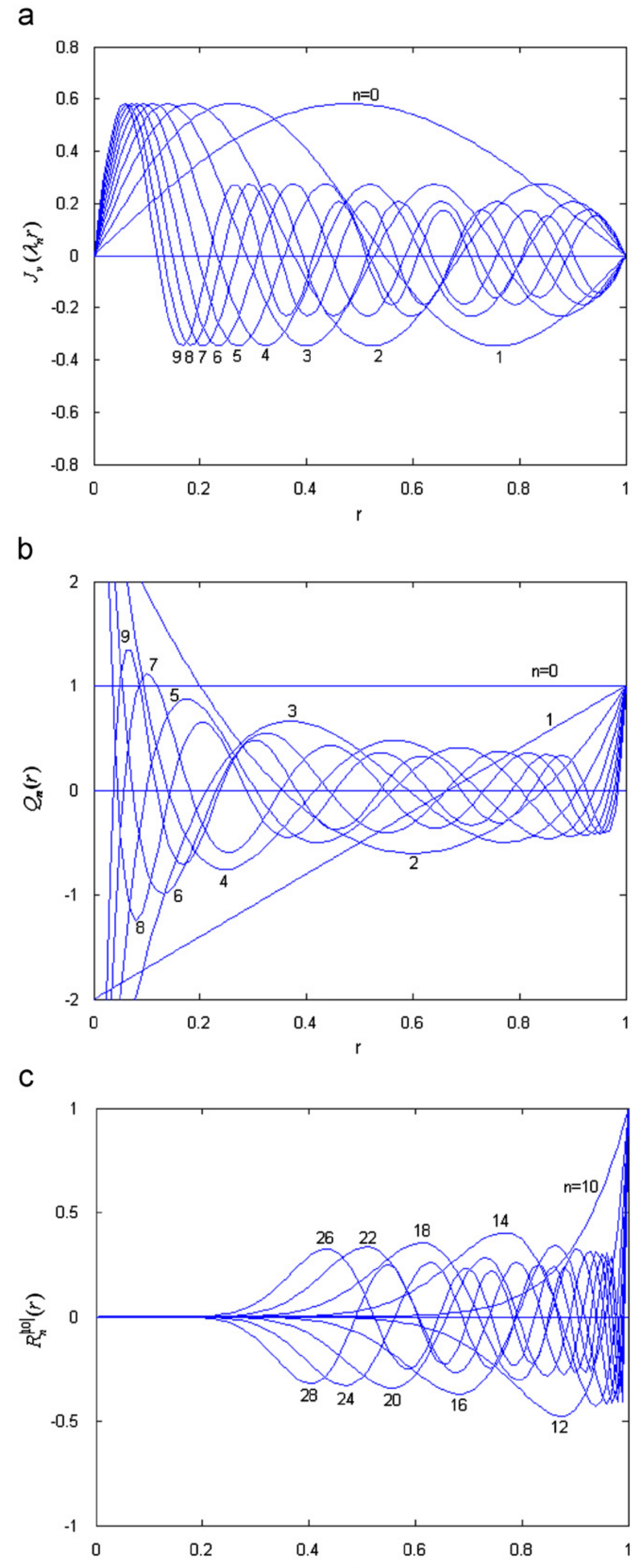

Fig. 1. (a) Radial polynomial $J_1(\lambda_n r)$ of the Bessel–Fourier moments with $n=0,1,\ldots,9$; (b) radial polynomial $Q_n(r)$ of the orthogonal Fourier–Mellin moments with $n=0,1,\ldots,9$ and (c) Zernike radial polynomial $R_n^{|m|}(r)$ with $m=10$ and $n=10,12,\ldots,28$.

where δ_{nk} is the Kronecker symbol, $r\in[0,1]$ is the size of the objects that can be encountered in a particular application. Hence the basis functions $J_v(\lambda_n r)\exp(-jm\theta)$ of the Bessel–Fourier