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Fractional subpixel diffusion and fuzzy logic approach for ultrasound speckle reduction

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ABSTRACT

Speckle is the dominant source of noise in ultrasound imaging and is a kind of multiplicative noise. It is difficult to design a filter to remove speckle effectively. In this paper, a novel fuzzy subpixel fractional partial difference (FSFPD) for ultrasound speckle reduction is proposed. Euler–Lagrange equation acts as an increasing function of the fractional derivative's absolute value of the image intensity function. The fractional order partial difference is computed in the frequency and fuzzy domain with subpixel precision. We test the proposed method on both synthetic and real breast ultrasound (BUS) images. The comparisons of the experimental results show that the proposed method can preserve edges and structural details of ultrasound images well while removing speckle noise. In addition, the filtered images are assessed and evaluated by radiologists using double blind method. The results demonstrate that the discrimination rate of breast cancers has been highly improved after employing the proposed method.

1. Introduction

Ultrasound (US) imaging is popularly used for diagnosis in recent years due to its advantages such as no-radiation, portability and low cost [1–3]. However, it suffers from two major drawbacks: low-contrast and speckle noise [4–5]. Speckle is inherent in US images, and is modeled as spatially correlated multiplicative noise which satisfies non-Rayleigh distribution [6]. It is a granular pattern generated by constructive and destructive coherent interference of backscattered echo from the scatterers which are much smaller than the spatial resolution of medical ultrasound systems [7–8]. Speckle often carries some useful information. However, speckle is a dominant factor affecting the contrast and resolution of US images, and is often considered as noise which should be filtered out without losing the features of the image [9]. Furthermore, speckle reduction is an important preprocessing step for segmentation and classification.

In the last three decades, a considerable effort has been made to develop the filters which can remove speckle while preserve the features of US images. Lee [10–11] and Frost [12] are the local statistic filters which achieve a balance between homogeneous regions and non-homogeneous regions. Heavy filtering is applied to the homogeneous regions to remove noise while light one is applied to the non-homogeneous regions to preserve the edges or other features. Kuan [13–14] is a generalization of Lee filter. An optimal detector of lines in fully developed speckle was derived using a generalized likelihood radio test (GLRT) [15]. A set of directional line-matched masks were used to extract the local data along different directions. It described optimal and sub-optimal approaches for detecting lines and boundaries from the images with speckle noise. However, the optimal detectors are computationally expensive, and the suboptimal detectors of linear and quadratic order are discussed. Some methods based on wavelet were developed to retain nonstationary signals from noise [16–18]. A robust wavelet method for noise filtering was proposed [16]. The method employed a preliminary detection of the wavelet coefficients representing the features of interest. Then it empirically estimated the probability density function of noise-free wavelet coefficients.

Recently, anisotropic diffusion methods have been studied as a useful tool for image noise removal. Anisotropic diffusion techniques were originally used for the generation of scale spaces [19]. Anisotropic diffusion is a nonlinear filtering method, which encourages diffusion in the homogeneous region whilst inhibiting diffusion at edges. In particular, anisotropic diffusion is often formulated in terms of a partial differential equation (PDE) which

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is discretized for handling digital images. Different discretization schemes may result in algorithms with different filter kernels [20]. A reference [21] proposed an edge-sensitive diffusion called speckle reducing anisotropic diffusion (SRAD). SRAD can preserve the edges by inhibiting diffusion across edges and allowing diffusion on either side of each edge.

However, most of existing speckle reduction techniques cannot truly and effectively enhance the edges. Actually, they only prevent smoothing near the edges [9]. Contrast enhancement is also difficult to implement for despeckle filters since speckle is usually described as non-Rayleigh distribution [22,23], and contrast enhancement often comes at the cost of resolution impairment [24].

In this paper, we present a novel anisotropic diffusion method, fuzzy subpixel fractional partial difference (FSFPD) method, for speckle reduction. Euler–Lagrange equation is used as an increasing function of the fractional derivative's absolute value of the image intensity function.

The paper is organized as follows. In Section 2, we review the fractional-order anisotropic diffusion. In Section 3, we introduce the proposed FSFPD algorithm. Section 4 compares the proposed approach with some published speckle reduction methods utilizing both synthetic and real ultrasound images. Finally, the conclusions are summarized in Section 5.

2. Fractional order anisotropic diffusion

Anisotropic diffusion is associated with an energy-dissipating process to seek the minimum of an energy function. The advantages of anisotropic diffusion include intra-region smoothing and edge preservation. For illustration purpose, the partial differential equation (PDE) of anisotropic diffusion is discussed in continuous domain first [19]. Let I denote the image, t the time, and $c(\cdot)$ the diffusion coefficient, then the anisotropic diffusion is formulated as

$$\frac{\partial I}{\partial t} = div(c(|\nabla I|)\nabla I) \tag{1}$$

where ∇ indicates the gradient operator with respect to the space variables, div is the divergence operator, and $c(|\nabla I|)$ is the diffusion coefficient.

Eq. (1) is associated with the energy functional to measure the oscillations in the image. According to the anisotropic diffusion theory, the energy function can be defined with different formulas [25]. Let I_0 be the observation of I with additive noise η. Noise is superimposed on the image and the resulting image is represented by $I_0 = I+\eta$. For instance, [26] proposed the functional

$$E(I) = \int_\Omega f(\alpha|\nabla I|_{\varepsilon 1} + \mu \frac{\ell(I)}{|\nabla I|^3_{\varepsilon 2}} + \frac{1}{2}(I-I_0)^2)\, d\Omega \tag{2}$$

where Ω is the image support, and $\ell(I)$ is an elliptic operator. The above functional is too complex and it costs more computing time, [11] proposed two different functionals to measure the oscillations in a noisy image

$$E_1(I) = \int_\Omega (|I_{xx}| + |I_{yy}|)\, d\Omega$$

$$E_2(I) = \int_\Omega (\sqrt{|I_{xx}|^2 + |I_{xy}|^2 + |I_{yx}|^2 + |I_{yy}|^2})\, d\Omega \tag{3}$$

The major difference between the above two functionals is that $E_2(I)$ is rotational invariant but $E_1(I)$ is not. Another functional was described [27]:

$$E(I) = \int_\Omega f(|\nabla I|)\, d\Omega \tag{4}$$

where $f(\cdot) \geq 0$ is an increasing function associated with the diffusion coefficient:

$$c(s) = f'(\frac{\sqrt{s})}{\sqrt{s}} \tag{5}$$

In this paper, an energy function is defined over a support of Ω [28]:

$$E(I) = \int_\Omega f(|D_\alpha I|)\, d\Omega \tag{6}$$

where $D_\alpha I = (D_{\alpha x} I, D_{\alpha y} I)$ and $|D_\alpha I| = \sqrt{D^2_{\alpha x} I + D^2_{\alpha y} I}$. The integer order α of the derivative can be generalized to a real number [29], and D_α is a fractional order derivative operator.

Euler–Lagrange equation can be constructed to seek the minimum of the energy function. For any function $\xi \in C^\infty(\Omega)$, a cost function $\phi(\lambda)$ is defined as

$$\Phi(\lambda) = \int_\Omega f(|D_\alpha I + \lambda D_\alpha \xi|)\, dx\, dy \tag{7}$$

where λ is a positive weight parameter. Then

$$\begin{aligned}\frac{d\Phi(0)}{d\lambda} &= \frac{d}{d\lambda}\int_\Omega f(|D_\alpha I + \lambda D_\alpha \xi|)\, dx\, dy|_{\lambda=0} \\ &= \int_\Omega (c(|D_\alpha I|^2) D_{\alpha x} I) D_{\alpha x}\xi\, dx\, dy \\ &\quad + \int_\Omega (c(|D_\alpha I|^2) D_{\alpha y} I D_{\alpha y}\xi\, dx\, dy)\end{aligned} \tag{8}$$

For all $\xi \in C^\infty(\Omega)$, $D^*_{\alpha x}$ and $D^*_{\alpha y}$ denote the Hermitian adjoints of operators $D_{\alpha x}$ and $D_{\alpha y}$, respectively. According to the property that, if A^* is the Hermitian adjoint of A, then $\langle Ax, y\rangle = \langle x, A^* y\rangle$, therefore, Eq. (8) can be rewritten as

$$\frac{d\Phi(0)}{d\lambda} = \int_\Omega (D^*_{\alpha x} c(|D_\alpha I|^2) D_{\alpha x} I)\xi\, dx\, dy + \int_\Omega (D^*_{\alpha y} c(|D_\alpha I|^2) D_{\alpha y} I)\xi\, dx\, dy \tag{9}$$

Thus, the Euler–Lagrange equation is

$$D^*_{\alpha x}(c(|D_\alpha I|^2) D_{\alpha x} I) + D^*_{\alpha y}(c(|D_\alpha I|^2) D_{\alpha y} I) = 0 \tag{10}$$

The Euler–Lagrange equation can be solved through the following gradient descent procedure:

$$\frac{\partial I}{\partial t} = -D^*_{\alpha x}(c(|D_\alpha I|^2) D_{\alpha x} I) - D^*_{\alpha y}(c(|D_\alpha I|^2) D_{\alpha y} I) \tag{11}$$

An important property of the fractional derivative is that, if $F(w)$ is the Fourier transform of $f(t)$, then the Fourier transform of the αth order derivative $D_\alpha f(t)$ is computed by $(jw)^\alpha F(w)$. For any function $f(t) \in L(R)$, its Fourier transform is

$$F(w) = \int_R f(t)\exp(-jwt)\, dt \tag{12}$$

For any $f(x,y) \in L^2(R^2)$, the 2-D Fourier transform is

$$F(w_1, w_2) = \int_{R^2} f(x,y)\exp(-j(w_1 x + w_2 y))\, dx\, dy \tag{13}$$

Fractional order partial derivative can be defined as [28]

$$D_{\alpha x} f = F^{-1}((1-\exp(-j2\pi w_1/M))^\alpha \times \exp(j\pi\alpha w_1/M) F(w_1, w_2)) \tag{14}$$

$$D^*_{\alpha x} f \leftrightarrow conj((1-\exp(-j2\pi w_1/M))^\alpha \times \exp(j\pi\alpha w_1/M)) F(w_1, w_2) \tag{15}$$

where F^{-1} denotes the inverse 2-D Fourier transform operator, and f is the continuously interpolated version of image I whose size is $M \times M$. Similarly, $D_{\alpha y} f$ and $D^*_{\alpha y} f$ can be derived.

3. Proposed approach

The behavior of the above anisotropic diffusion approach is still dependent on the shape of the energy surface [30]. The anisotropic diffusion approach bears some fuzziness due to the