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Image reconstruction from limited range projections using orthogonal moments

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Abstract

A set of orthonormal polynomials is proposed for image reconstruction from projection data. The relationship between the projection moments and image moments is discussed in detail, and some interesting properties are demonstrated. Simulation results are provided to validate the method and to compare its performance with previous works.

Keywords: Image reconstruction; Radon transform; Projection moments; Image moments; Orthonormal polynomials

1. Introduction

In its classical formulation, computerized tomography (CT) deals with the reconstruction of an object from measurements which are line integrals of that object at some known orientations. This formulation has found many applications in the fields of medical imaging, synthetic aperture radar, electron-microscopy-based tomography, etc. [1]. The radon transformation, due to its explicit geometric meaning, has played an important role.

In the past decades, a number of studies have been carried out on this subject. Lewitt [2] has summarized a series of projection theorems, a detailed description about the properties of radon transform and its relationship to other transforms have been given by Deans [1]. Major contributions have been reported in the biomedical engineering literature, one of the most active fields for the reconstruction problem, in order to study new X-ray source–detector trajectories (typically a spiral for CT) and to deal with truncated [3] and cone-beam projections [4].

Recently, the moment-based approaches to tomographic reconstruction have attracted considerable attention of several research groups. Salzman [5] and Goncharev [6], respectively, proposed the methods based on the moments to find the view angle from the projection data. Milanfar et al. [7] described a variational framework for the tomographic reconstruction of an image from the maximum likelihood estimates of its orthogonal moments. Basu and Bresler [8,9] discussed the problem of recovering the view angles using moments of the projections. Wang and Sze [10] proposed an approach based on the relationship between the projection moments and the image moments to reconstruct the CT images from limited range projections. In Wang's algorithm, the geometric moments were used and interesting results have been obtained. However, the use of the geometric moments has the following disadvantages: (1) the geometric moments of an image are integrals of the field shape over space, and the image can be uniquely determined by the geometric moments of all orders. They are sensitive to digitization error and minor shape deformations [11]; (2) the geometric moments are basically projections of the image function onto the monomials $x^n y^m$. Unfortunately, the basis set $\{x^n y^m\}$ is not orthogonal. These moments are therefore not optimal with regard to the information redundancy and

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other useful properties that may result from using orthogonal basis functions.

To overcome these inconveniences, we propose in this paper, by extending Wang's algorithm, a new moment-based approach using the orthogonal basis set to reconstruct the image from limited range projection.

The paper is organized as follows. A brief review of radon transform and the definition of projection moments and image moments are given in Section 2. In Section 2, we also establish the relationship between projection moments and image moments and discuss how to estimate the projection moments at any specific view from image moments. Simulation results are provided in Section 3. Section 4 concludes the paper and provides some additional perspectives.

2. Method

We first sketch the basics of radon transform. The orthogonal projection moments, defined in terms of normalized polynomials, are then introduced in Section 2.2. Some theorems relating projection and image moments are reported and demonstrated in Section 2.3. In the last subsection, we show how to estimate the unknown projections from the calculated image moments.

2.1. Radon transform

Let $f(x, y) \in L^2(D)$ be a square-integrable function with support inside the unit circle D in the plane and $g(s, \theta)$ be the radon transform of $f(x, y)$ defined as follows:

$$g(s, \theta) = \int\int_D f(x, y)\delta(x\cos\theta + y\sin\theta - s)\,dx\,dy, \quad (1)$$

where $\delta(\cdot)$ denotes the Dirac delta function, s is the distance from the origin to the ray, and θ is the angle between the x-axis and the ray.

For a given view θ, the 2D function $g(s, \theta)$ becomes a one-variable of s, denoted by $g_\theta(s)$. Since $g_\theta(s)$ represents a collection of integrals along a set of parallel rays, it is also called parallel projection of $g(s, \theta)$ at view θ [10]. The radon transform given by Eq. (1) can be rewritten as

$$g_\theta(s) = \int\int_D f(x, y)\delta(x\cos\theta + y\sin\theta - s)\,dx\,dy. \quad (2)$$

2.2. Orthogonal projection moments and image moments

The moments of $g_\theta(s)$ are called projection moments in the radon domain [2]. In this paper, we use a set of orthonormal polynomials instead of the set of monomials $\{s^p\}$ to define the projection moments. Let $\{P_p(s)\}$, $p = 0, 1, 2, \ldots\infty$, be a set of orthonormal polynomials defined on the interval $[-1, 1]$, the pth order orthonormal projection moment of $g_\theta(s)$ is defined as

$$L_p(\theta) = \int_{-1}^{1} P_p(s)g_\theta(s)\,ds. \quad (3)$$

Let λ_{nm} be the $(n+m)$th order orthogonal moment of $f(x, y)$ defined as [12]

$$\lambda_{nm} = \int\int_D P_n(x)P_m(y)f(x, y)\,dx\,dy. \quad (4)$$

Substitution of Eq. (2) into Eq. (3) yields

$$\begin{aligned} L_p(\theta) &= \int_{-1}^{1}\int\int_D f(x, y)\delta(x\cos\theta \\ &\quad + y\sin\theta - s)P_p(s)\,dx\,dy\,ds \\ &= \int\int_D\int_{-1}^{1} f(x, y)\delta(x\cos\theta \\ &\quad + y\sin\theta - s)P_p(s)\,ds\,dx\,dy. \end{aligned} \quad (5)$$

Using the property of delta function, we have

$$L_p(\theta) = \int\int_D P_p(x\cos\theta + y\sin\theta)f(x, y)\,dx\,dy. \quad (6)$$

This last equation will allow us to establish a relationship between the orthogonal projection moments defined by Eq. (3) and the orthogonal moments of $f(x, y)$ defined by Eq. (4). This is the objective of the next subsection.

2.3. Relationship between projection moments and image moments

Let us first introduce some basic definitions. Let the pth order normalized polynomials $P_p(t)$ be

$$P_p(t) = \sum_{r=0}^{p} c_{pr}t^r \quad (7)$$

and let $V_p(t) = (P_0(t), P_1(t), P_2(t), \ldots, P_p(t))^T$ and $M_p(t) = (1, t, t^2, \ldots, t^p)^T$ where the superscript T indicates the transposition, then we have

$$V_p(t) = C_pM_p(t), \quad (8)$$

where $C_p = (c_{kr})$, with $0 \leqslant r \leqslant k \leqslant p$, is a $(p+1) \times (p+1)$ lower triangular matrix.

Since all the diagonal elements of C_p, c_{kk}, are not zero, the matrix C_p is non-singular, thus

$$M_p(t) = C_p^{-1}V_p(t) = D_pV_p(t), \quad (9)$$

where $D_p = (d_{kr})$, with $0 \leqslant r \leqslant k \leqslant p$, is the inverse matrix of C_p.

Eq. (9) can be rewritten as

$$t^k = \sum_{r=0}^{k} d_{kr}P_r(t) \quad \text{for } 0 \leqslant k \leqslant p. \quad (10)$$