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Fast computation of geometric moments using a symmetric kernel

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Abstract

This paper presents a novel set of geometric moments with symmetric kernel (SGM) obtained using an appropriate transformation of image coordinates. By using this image transformation, the computational complexity of geometric moments (GM) is reduced significantly through the embedded symmetry and separability properties. In addition, it minimizes the numerical instability problem that occurs in high order GM computation. The novelty of the method proposed in this paper lies in the transformation of GM kernel from interval $[0, \infty]$ to interval $[-1, 1]$. The transformed GM monomials are symmetry at the origin of principal Cartesian coordinate axes and hence possess symmetrical property. The computational complexity of SGM is reduced significantly from order $O(N^4)$ using the original form of computation to order $O(N^3)$ for the proposed symmetry-separable approach. Experimental results show that the percentage of reduction in computation time of the proposed SGM over the original GM is very significant at about 75.0% and 50.0% for square and non-square images, respectively. Furthermore, the invariant properties of translation, scaling and rotation in Hu's moment invariants are maintained. The advantages of applying SGM over GM in Zernike moments computation in terms of efficient representation and computation have been shown through experimental results.

Keywords: Geometric moments with symmetric kernel (SGM); Fast computation; Symmetrical property; Numerical instability; Invariant properties; Zernike moments; Efficient representation; Computation

1. Introduction

In the past few decades, moment functions have received considerable attention in the field of image processing since the first application of geometric moments (or regular moments) in invariant visual pattern recognition introduced by Hu [1]. Moment functions of a two-dimensional digital image can be used to transform the image into a feature space where the transformed coefficients represent certain shape characteristics of the image. Fundamentally, moment functions can be categorized into two main groups, depending on the domain of definition of the polynomial subspace. The polynomial subspace of the first group of moment functions is defined over the Cartesian space while for the second group of moment functions, it is defined over the polar coordinate space.

Geometric (GM), Legendre (LM), Tchebichef (TM) and Krawtchouk (KM) moments are subsumed to the first group of moment functions while complex, rotational, Zernike and pseudo-Zernike moments are subsumed to the second group. Mathematically, the polar coordinates can be represented in terms of Cartesian coordinates. Hence, all the aforementioned polynomials can be defined within the same space (Cartesian space) after a proper coordinate transformation. Since different sets of polynomials up to the same order are defined in the same subspace, any complete set of moments up to a given order can be obtained from any other set of moments up to same order, at least in theory [2]. Hence, most of the moment functions can be computed exactly or approximately from the simple GM.

The typical applications of GM in image analysis include pattern recognition or classification [3,4], scene matching [5], edge detection [6], orientation determination [7], attitude estimation [8], texture analysis [9], shape analysis [10], image registration [11], watermarking [12], face expression recognition [13] and content-based image retrieval [14]. Other moment

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functions defined in Cartesian coordinate system such as LM [15], TM [16], and KM [17] moments have recently attracted the attention of researchers in image analysis. The readers can refer to Ref. [18] for details on properties of various moment functions and some of their applications in image analysis.

Notwithstanding the usefulness of GM in image analysis, the applications of GM in real time are limited by its computational complexity especially when the image resolution and/or moment order is large. This motivates researchers to explore fast and efficient computational methods to minimize the complexity and hence increase their effectiveness and feasibility in real time applications. Efficient algorithms which were proposed to enhance the computational speed include the delta method [19], integration method [20], corner point method [21], Green's Theorem-based method [22], Discrete Radon Transform-based method [23] and accumulation moments-based method [24]. Due to algorithmic constraints, these methods are not optimal and only feasible in limited conditions. Approximation error also occurs during the GM computation that is performed using representation in the domain other than monomials.

The constraints which limit the feasibility of the aforementioned algorithms are listed below:

- delta and integration methods are only applicable for binary images [19], [20];
- corner points method is only suitable for binary images [19] with simple object shape [21];
- discrete Green's theorem based method is only suitable for binary images [22];
- discrete Radon transform based method requires the projection of data through cyclic operations which are other than monomials domain [23];
- accumulation moments based method could cause an underlying numerical ill-conditioning to appear in the inverse transform of high order GM due to roundoff error [24].

The efficient computation of orthogonal moment functions such as LM, TM, and KM is achieved normally using the recurrence relationships between their successive polynomials [16–18]. The respective moment polynomials are computed recursively using a two-term recurrence formula where the initial condition, zeroth and first order polynomials are computed directly. This recursive computation together with the separability property successfully reduces the computational complexity of the moment functions from order $O(N^4)$ to order $O(2N^3)$. Moment functions with its polynomials defined within interval $[-1, 1]$ possess symmetrical property as shown in radial moment functions [25,26]. Due to similar definition in their individual polynomials, LM, TM, and KM also possess the symmetrical property and their computational complexity is further reduced to order $O(N^3)$. This significant reduction in computation complexity from order $O(N^4)$ to order $O(N^3)$ increases the usability of the moment functions in real world applications. The combination of recursive computation with separability and symmetrical properties to reduce the computational complexity in LM, TM, KM, and radial moments prompted the authors to apply a similar approach for the fast computation of GM.

In this paper, a novel set of GM is proposed after an appropriate coordinate transformation in order to achieve the fast computation of GM. In this transformation, GM monomials are projected within the interval of $[-1, 1]$ instead of $[0, \infty]$. This enables the monomials to possess the symmetrical property as Legendre polynomials. The added advantage is the minimization of the numerical instability problem during the computation of high order GM from large resolution images. This novel set of GM is denoted as geometric moments with symmetric kernel (SGM). By including the symmetrical property in current recursive computation method, the computational complexity of SGM is reduced significantly from order $O(N^4)$ to order $O(N^3)$ which is similar in the LM, TM, KM, and radial moments computation.

The organization of this paper is as follows. The basic theories of moment functions including the GM and LM are provided in Section 2. Section 3 presents the polynomials properties which allow the reduction in computational complexity of moment functions. Section 4 provides the details on the symmetrical property of LM and how it can be used to derive a novel set of GM denoted as SGM. Some properties of the proposed SGM are also discussed in this section. The fast computation of the proposed SGM using the symmetrical and separability properties through matrix implementation is described in Section 5. The definition of Zernike moments (ZM) and the application of SGM in exact ZM computation are presented in Section 6. In Section 7, the performances of the proposed SGM and the original GM in terms of computational speed are compared through experiments. The advantages of using the proposed SGM over GM in exact ZM computation are validated through experiments. Finally, Section 8 concludes this study.

2. Moment functions

The general definition of moment functions as the correlation measure between two functions is presented in this section. Then, the brief definition and properties of GM and LM are provided.

2.1. Moment functions as correlation measures

Correlation is a similarity measure between functions [27]. The correlation measure between two functions $r(x, y)$ and $s(x, y)$ is defined as

$$M = \int_0^\infty \int_0^\infty r(x, y)[s(x, y)]^* \,\mathrm{d}x\,\mathrm{d}y, \tag{1}$$

where $(\cdot)^*$ denotes the complex conjugate. Eq. (1) can be used to measure the similarity between moment polynomials and two-dimensional image if $s(x, y)$ is replaced by the moment polynomials and $r(x, y)$ is replaced by the image intensity function, respectively. Hence, the general definition of $(p+q)$th order moment is defined as [2]

$$M_{pq} = \lambda_{pq} \int_0^\infty \int_0^\infty f(x, y)[V_{pq}(x, y)]^* \,\mathrm{d}x\,\mathrm{d}y, \tag{2}$$