Pattern Recognition 60 (2016) 720–733

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Fuzzy mathematical morphology for color images defined by fuzzy preference relations

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ARTICLE INFO

Article history:
Received 2 July 2015
Received in revised form
25 May 2016
Accepted 17 June 2016
Available online 27 June 2016

Keywords:
Mathematical Morphology
Color Images
Morphological Operators
Fuzzy Order
Fuzzy Preference Relations

ABSTRACT

Nowadays, the representation and the treatment of color images are still open problems. Mathematical morphology is the natural area for a rigorous formulation of many problems in image analysis. Moreover, it comprises powerful non-linear techniques for filtering, texture analysis, shape analysis, edge detection or segmentation. A large number of morphological operators have been widely defined and tested to process binary and gray images. However, the extension of mathematical morphology operators to multivalued functions, and in particular to color images, is neither direct nor general due to the vectorial nature of the data. In this paper, basic morphological operators, erosion and dilation, are extended to color images from a new vector ordering scheme based on a fuzzy order in the RGB color space. Experimental results show that the proposed color operators can be efficiently used for color image processing.

1. Introduction

Techniques of artificial vision have been initially developed for binary and gray scale images, where the information is codified by 2 and 2^{n+1} with $n \in \mathbb{N}$ levels respectively. Nevertheless, the color is an important source of information. For this reason, during the last years these techniques have been developed for color images. However, nowadays, the representation and the treatment of color images are still open problems [1–4].

Mathematical morphology is the natural area for a rigorous formulation of many problems in image analysis, as well as a powerful non-linear technique which includes operators for the filtering, texture analysis, shape analysis, edge detection or segmentation. In the 1980s, Matheron and Serra [5–8] proposed the last mathematical formulation of morphology within the algebraic framework of the lattices. This means that the definition of morphological operators needs a totally ordered complete lattice structure. In that context, before defining the basic morphological operators (erosion and dilation) it is necessary to define an order on the space used for processing the images.

The definition of an order for both binary and gray levels images is straightforward because for both sets an order relation exists, inclusion for binary images and the order relation inherited from $\mathbb{R}$ for gray levels image. However, for color images two problems arise. On one hand, the chromatic space in which the image is processed [9,10]. On the other hand, it is not clear what order is the best because of the vectorial nature of the data [11–19]. Barnett [20] introduced four types of vector orderings: marginal (M-order), reduced (R-order), conditional (C-order) and partial (P-order). When applied to color data, all these orderings have certain disadvantages, depending on the goal. For instance, the marginal ordering introduces false colors [16,21,22] and the conditional ordering generates visual nonlinearities from the human perception point of view; the reduced and partial orderings are either relying on pre-orderings, thus lacking the anti-symmetry property, or generate perceptual nonlinearities, as conditional orderings.

A widely used order in the literature is the lexicographic one. Several authors have used the complete lattice associated to this order and defined the dilation and erosion operators for color images according to it [1,10–12,18,21]. Although the results based on it are successful, this order is not suitable because some component is more important than the others when vectors are sorted. Therefore, it is necessary to make a choice in advance and therefore the component selected as priority will have more weight

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http://dx.doi.org/10.1016/j.patcog.2016.06.014
0031-3203/

than the others, so that the order will depend on this choice. This is an important drawback because in many problems all the components must have the same importance.

The starting point of this paper is fuzzy mathematical morphology. It is a different approach extension of the mathematical morphology's binary operators to gray level images, by redefining the set operations as fuzzy set operations. It is based on fuzzy sets theory [23–30]. The goal of this paper is to define the operators of the fuzzy mathematical morphology for color images through the use of a fuzzy order. In addition, the extended operators consider all the components with the same weight and avoid false colors.

Note that this approach could be applied to other multivariate images. However, the extension of mathematical morphological operators to multichannel data with hundreds of spectral channels is not straightforward. Some interesting results about ordering for multivariate images in high dimensional spaces can be found in [31–33]. The approach proposed in [31] consists in computing an order based on the distance to a central value, obtained by the statistical depth function, while in [32] an additive morphological decomposition based on morphological operators is considered. On the other hand, in [33], the authors presented different kinds of partial orders based on the end member representation of the hyperspectral images.

This paper is organized as follows: Section 2 reviews the general concepts of the mathematical morphology for color images; Section 3 presents the main contribution of this paper, i.e., a fuzzy ordering based on preference relations. Besides, the way in which the fuzzy order is applied to color images is explained. Section 4 studies invariant properties. In Section 5 the results obtained with the fuzzy mathematical morphology for some color images are shown. Finally, the last section details some conclusions and discusses possible future lines of research.

2. Mathematical morphology for color images

Mathematical morphology (MM) is a non-linear theory for spatial analysis of images where the topological relations and the geometry of the objects in the image are the parameters characterizing the object under study [5–7,34]. The main idea of this methodology is the decomposition of a operator into a combination of the basic operators: erosion, dilation, anti-erosion and anti-dilation, as well as the supreme and the infimum operations.

Fuzzy set theory has been extensively applied to image processing [27]. In particular, one extension of binary MM is the fuzzy mathematical morphology (FMM) [23,24,27–30,35]. It incorporates fuzzy logic concepts for extending binary morphological operators to gray levels images allowing us to model and manipulate in a different way the uncertainty and imprecision present almost in all different types of images. One possible approach is to fuzzify the logical operators involved in the intersection and inclusion definitions required to define dilation and erosion [23,24]. In the same line in [27] fuzzy dilation and fuzzy erosion is presented a similar approach but t-norms are used instead of a conjunctor to define the fuzzy dilation and the associated model implicator to define the fuzzy erosion. This model is less general than the previous one. Extending the Minkowski addition to an operation on fuzzy sets in $\mathbb{R}^n$ is another possibility to define a fuzzy dilation [24]. There are also other different approaches as those based on non-fault operators [36].

The afore mentioned approaches extend the initial basic morphological operators defined for sets (binary images) in an immediate way to functions (gray levels images) [5–7,34,37]. This extension was made in a natural way because in both sets exist an order relation, inclusion for binary images and the order relation inherited from $\mathbb{R}$ in the case of gray level images. Therefore, the complete lattice can integrate the theoretical models of the MM. Assuming this, the color mathematical morphology (CMM) can be developed from the MM for gray level images. In that case, the definition of a complete lattice in the color space representing the chromatic information of the digital images becomes necessary. However, there is not a natural order for multidimensional data and therefore the extension of MM to CMM is not straightforward.

This will be our starting point for processing color images. Analogously to CMM, the FMM for color images is developed from the definition of a total fuzzy order.

The morphological processing of color images, modeled as functions $f: D_f \subset \mathbb{R}^2 \to \tau \subset \mathbb{R}^3$ (D_f is the domain of the image and τ represents the color space) is usually performed from two points of view: the marginal processing and the vectorial processing [13].

The marginal processing consists in applying the morphological operators defined for gray scale images to each color component of the image. An important drawback of this approach is that false colors usually appear as the combination of the processing components that generate new sequences of pixels. Actually this is an important problem that should be avoided. The vectorial processing is based on applying a unique operation to the image considering it as an indivisible composition of vectorial pixels. In such approach the notion of a complete lattice arises and therefore the definition of a total order over the subset τ of $\mathbb{R}^3$ becomes necessary. Since there is not any natural order in these sets, it is necessary to establish an appropriate order on color space τ.

As marginal processing is a particular case of vectorial processing, once the order is defined, it is possible to provide a general definition for the basic operators called erosion and dilation (see Section 2.2). Previously, it is necessary to introduce the concept of a structuring element.

2.1. Structuring element

MM examines the geometrical structures of the image by checking a small neighborhood, called structuring elements (SEs), in different parts of the image. The SE is a completely defined set whose geometry is known in advance. It is compared to the image through translations. The size and shape of the SEs are chosen a priori depending on the morphology of the set over which it interacts and also according to the desired shape extraction. The SEs are then moved, so that they cover the whole image pixel by pixel, making a comparison between each element and the image.

To define the operators of the CMM, the SE plays an important role. In this case, the SE is not a set as in the binary case or a function as in the case of gray levels images, but it indicates the neighborhood over which the pixels of the image are compared.

Definition 1. Let $f: D_f \subset \mathbb{R}^2 \to \mathbb{R}^3$ be a color image, let $x \in D_f$ be a pixel, and let (D_f, d) be a metric space. A *structuring element* (SE for short) for a color pixel x is a neighborhood:

$$B(x, r) = \{y \in D_f: d(x, y) \le r\} \tag{1}$$

where r denotes any positive real number, which is called *diameter*.

Remark 2. The function d is a metric or distance function on D_f, that is, for any $x, y, z \in D_f$, d satisfies the following properties:

i. $d(x, y) \ge 0$,
ii. $d(x, y) = 0 \Leftrightarrow x = y$,
iii. $d(x, y) = d(y, x)$,
iv. $d(x, y) \le d(x, z) + d(z, y)$.