Pattern Recognition 60 (2016) 657–668

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Symmetry-aware graph matching

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ARTICLE INFO

Article history:
Received 3 December 2015
Received in revised form
18 April 2016
Accepted 25 June 2016
Available online 27 June 2016

Keywords:
Symmetry
Graph matching
Image matching
Geometric information

ABSTRACT

Visual symmetry encodes middle- to high-level geometric information and plays an important role in many computer vision applications. Not much effort, however, has been devoted to utilize symmetry information for graph matching. In this paper, we propose a new framework for symmetry-aware graph matching in the context of image matching. In the framework, we first define symmetry matrices to characterize the geometric symmetry relation among image features, and then develop two methods to discover such symmetry. After that we design general strategies to boost graph matching algorithms with symmetry constraints, and apply these strategies to several state-of-the-art algorithms. For evaluation, the proposed symmetry-aware graph matching algorithms are first applied to two datasets involving clear symmetry patterns: a recently proposed architecture image set and a dental panoramic radiograph collection. The results clearly demonstrate the benefits of using symmetry information. An experiment is also conducted on a dataset where symmetry is less obvious, the results show that our strategies still achieve better of similar performance compared with state-of-the-arts.

1. Introduction

Graph matching involves establishing correspondences between the vertices of two graphs. As a fundamental problem in computer science, graph matching closely relates to a lot of research problems in computer vision, including for instance feature registration [1], image classification [2,3], object recognition [4] and shape matching [5]. Graph matching is in general NP-hard, and global optimum is hardly guaranteed for graphs of reasonable sizes. Consequently, approximate algorithms seeking an acceptable suboptimal solutions are popular and task-dependent constraints are often utilized to harness the solution searching.

Matching problems in computer vision often require constraints over vertices in the graph. For instance, geometric constraints like affine transformation are often required in the matching points from two images. Handling this issue, prior knowledge, such as global geometric transformations [7], spatial layouts [8,9] and triplets affinities [10,11], is frequently encoded into graph matching. Despite these studies, little efforts have been devoted to integrate symmetry information for graph matching.

As a cornerstone of nature, symmetry refers to self-similarity or invariance exhibiting oneself through the shapes of natural creations and immanent laws of geometry [12]. In addition to natural scenes, symmetry also exists pervasively in manmade objects ranging from cups to airplanes. It is as well known that symmetric patterns are often salient for human visual attention. Particularly, symmetry is a potentially stable and robust high-level geometric feature of images invariant to scale, rotation and translation. These characteristics motivate us to integrate symmetry into graph matching to improve its performance in computer vision. Fig. 1 illustrates an example of image matching *without* versus *with* our proposed symmetry encoding, where we can see clear improvement by using symmetry information.

In this paper, we propose symmetry constraints for improving graph matching in the context of image matching. We develop a general framework for this purpose and validate the idea using several state-of-the-art graph matching algorithms as baselines. In particular, we first define symmetry matrix to characterize graph symmetry,[1] and then develop two methods to discover or measure such symmetry from image features. After that, we propose two general strategies to integrate symmetry constraints into graph matching algorithms.

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[1] By *graph symmetry* we mean the spatial symmetry of graph vertices in the image coordinate system, not necessarily in graph topology.

http://dx.doi.org/10.1016/j.patcog.2016.06.022
0031-3203/

Fig. 1. An example of image matching. Top: the GNCCP algorithm [6] without using symmetry constraints. Bottom: the same algorithm boosted by our symmetry integration strategy. Green and red lines indicate respectively correct and wrong matches – same patterns are used for Figs. 6–8 and are best viewed in color. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

To validate the general idea of using symmetry information, we apply the proposed framework to fuse symmetry constraints into six recently proposed graph matching algorithms [13–17,6]. For evaluation, these algorithms are first applied to two datasets involving clear symmetry patterns: a recently proposed architecture image set [18] and a dental panoramic radiograph collection. The results show that, by integrating symmetry constraints, we can consistently improve the matching performance across different baselines. For further understanding, these algorithms are also conducted on a dataset [15] where symmetry is less obvious, the results show that our strategies still achieve better or similar performance compared with state-of-the-arts.

This paper makes several contributions. (1) We introduce symmetry constraints into graph matching and propose a general framework to accommodate such constraints. To the best of our knowledge, this is the first work on integrating symmetry constraints for graph matching. (2) We define a symmetry matrix to characterize graph symmetry and develop two algorithms for symmetry discovery. (3) We demonstrate how to integrate symmetry constraints into various graph matching algorithms. (4) We construct a new real world dataset with annotation for evaluating graph matching algorithms.

The rest of this paper is organized as follows: In Section 2, we review some related works on graph matching and symmetry analysis. We present a general framework for symmetry-aware graph matching in Section 3, and two methods of symmetry discovery in Section 4. After that, we discuss how to fuse symmetry constraints into existing graph matching algorithms in Section 5. Some implemental details are presented in Section 6 and experimental results in Section 7. Finally, we draw the conclusions in Section 8.

2. Related works

2.1. Graph matching in computer vision

Graph matching has been investigated for four decades, and a large amount of algorithms have been proposed for matching graphs and sets of points. Two comprehensive surveys are reported in [19,20]. In general, graph matching has a combinatorial nature and there is no known efficient algorithm for global optimum. It is therefore commonly solved using approximate solutions or under relaxed conditions or constraints. Thoroughly reviewing all graph matching papers is beyond the scope of this paper, in the following we sample some related ones that inspire our study.

A popular way to find approximate graph matching solutions is based on spectral relaxation. Leordeanu and Hebert [21] present an efficient and robust solution by spectral relaxation that ignores the assignment constraints and is solved by eigen-analysis. The assignment constraints are then enforced during the discretization step. Cour et al. [22] extended [21] by first encoding affine constraints into the spectral decomposition and then applying bistochastic normalization to balance the affinity matrix. Emms et al. [23] reformulate graph matching as a problem of continuous-time quantum walk on an auxiliary graph structure. Cho et al. [15] reformulate graph matching as a vertex selection problem and introduce an affinity-preserving random walk algorithm. The algorithm is proved to be equivalent to the spectral relaxation [21] for the *integer quadratic programming* (IQP) formulation.

While graph matching is inherently a discrete optimization problem, another group of approximate algorithms reformulate it in the continuous domain by relaxing related constraints. The continuous solution is later discretized towards the final solution. For instance, Leordeanu et al. [13] propose an integer projection algorithm to optimize the objective function in an integer domain. Zaslavskiy et al. [14] reformulated graph matching as a *convex–concave relaxation procedure* (CCRP) problem that is solved by interpolating between two approximate simpler formulations. Zhou and Torre [16] apply the similar strategy, and factorize an affinity matrix into a Kronecker product of smaller matrices, each of them encodes the structure of the graphs and the affinities between vertices and between edges. Jiang et al. [24] follow the factorization scheme and propose a sparse model for graph matching. Liu and Qiao [6,25] proposed the *graduated nonconvexity and concavity procedure* (GNCCP) to equivalently realize CCRP on partial permutation matrix.

Another line of algorithms is largely inspired by Zass and Shashua [26], where a probabilistic framework is employed for (hyper) graph matching. The work introduces the following two assumptions: (1) the pairwise affinity matrix is an empirical estimate of the