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Generic orthogonal moments: Jacobi–Fourier moments for invariant image description

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Abstract

A multi-distorted invariant orthogonal moments, Jacobi–Fourier Moments (JFM), were proposed. The integral kernel of the moments was composed of radial Jacobi polynomial and angular Fourier complex componential factor. The variation of two parameters in Jacobi polynomial, α and β, can form various types of orthogonal moments: Legendre–Fourier Moments ($\alpha = 1, \beta = 1$); Chebyshev–Fourier Moments ($\alpha = 2, \beta = \frac{3}{2}$); Orthogonal Fourier–Mellin Moments ($\alpha = 2, \beta = 2$); Zernike Moments and pseudo-Zernike Moments, and so on. Therefore, Jacobi–Fourier Moments are generic expressions of orthogonal moments formed by a radial orthogonal polynomial and angular Fourier complex component factor, providing a common mathematical tool for performance analysis of the orthogonal moments. In the paper, Jacobi–Fourier Moments were calculated for a deterministic image, and the original image was reconstructed with the moments. The relationship between Jacobi–Fourier Moments and other orthogonal moments was studied. Theoretical analysis and experimental investigation were conducted in terms of the description performance and noise sensibility of the JFM.

Keywords: Jacobi polynomial; Multi-distorted invariance; Jacobi–Fourier Moments; Image reconstruction error; Noise sensibility

1. Introduction

There are a lot of literature on distorted-invariant pattern recognition including distorted-invariant descriptor [1–4] and synthetic filter method [15,16]. Those methods have solved the problem of some aspect of distorted invariant pattern recognition. It is important to acquire a set of orthogonal and multi-distorted invariant features of an image for image description and multi-distorted invariant pattern recognition. Hu [5] derived the moment invariant from geometrical moments in 1962, which are invariant for translation, rotation and scaling of the image. Hu's moment invariants are not orthogonal themselves, so that reconstruction of the image with Hu's moment invariant is impossible. The reconstruction of an image is an important question on how many moment invariants should be used and how well they describe the image. According to the orthogonal theory [6], an image function can be decomposed with orthogonal and completed function systems to form the independent orthogonal image moments, and original image can be reconstructed by the weighted superposition of the moments. Quality of the reconstructed image and quantity of the orthogonal moments needed for reconstructing an image can be evaluated by the reconstruction process.

Teagure [7] first used the Zernike Moments [8] (ZM) for shape description. The kernel function of Zernike Moments is Zernike function system which is composed of a radial Zernike polynomial and an angular Fourier complex componential factor in polar coordinate system. Zernike

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Moments are orthogonal moments. Sheng et al. proposed the Orthogonal Fourier–Mellin Moments (OFMM) [9], which are constructed by the Gram–Schmidt orthogonization of a set of monomials of the lowest powers. Other orthogonal moments, such as Chebyshev–Fourier Moments (CHFM) [11], Radial-Harmonic-Fourier Moments (RHFM) [12] was proposed, as well. All the moments are constructed by the radial orthogonal polynomials and angular Fourier complex exponential factor to form the orthogonal kernel function to decompose the image function in polar coordinate system. Except this type of orthogonal moments, there are some other types of orthogonal moments, such as Legendre Moments (LM) [10], Discrete Chebyshev Moments (DCHM) [13] and Complex Moments (CM) [14] in a Cartesian coordinate system, but they are not rotation invariant.

Teh et al. commented the description performance and noise sensibility of various image moments in 1988. They found that Zernike Moments have a superior performance over the others [10]. Sheng et al. [9] showed that the OFMM possesses a better performance over the Zernike Moments in 1994, especially for the description of small images. Ping et al. [11] showed that Chebishev–Fourier Moments possess the same performance as that of the OFMM. Ren et al. [12] showed that the Radial-Harmonic-Fourier Moments have the best performance in terms of image reconstruction and noise sensibility.

Each of the above moments is independent and not associated with each other. We propose orthogonal moments, Jacobi–Fourier Moments (JFM), the kernel function of which consists of radial Jacobi polynomial and angular Fourier complex exponential factor. The JFM is a generic orthogonal moment. All orthogonal moments with the kernel function consisted of radial orthogonal polynomial and angular Fourier exponential factor are special cases of the JFM. A common formulation of orthogonal moments will benefit the performance analysis of the moments.

In Section 2, the definition of the JFM is given and the behavior of Jacobi polynomials near the origin point of the coordinate system is investigated. In Section 3, the multi-distortion invariance of JFM is discussed and the JFM is normalized for scale and intensity distorted invariance. In Section 4, JFM of English capital alphabet is calculated with various types of JFM and the original image is reconstructed with the moments, and the performance of JFM is analyzed in terms of image reconstruction error and noise sensibility. The last section is the conclusion. The relationship between JFM and other orthogonal moments is proved in the Appendix.

2. JFM

The JFM kernel function set $P_{nm}(r,\vartheta)$ consists of two separable function sets: the deformed Jacobi polynomial $J_n(\alpha,\beta,r)$ to be a radial function and the Fourier exponential factor $\exp(\mathrm{j}m\vartheta)$ to be an angular function:

$$P_{nm}(r,\vartheta) = J_n(\alpha,\beta,r)\exp(\mathrm{j}m\vartheta), \tag{1}$$

where n and m are integers. The Jacobi–Fourier kernel function set is orthogonal in the interior of the unit circle, $0 \leqslant r \leqslant 1$, $0 \leqslant \vartheta \leqslant 2\pi$.

$$\int_0^{2\pi}\int_0^1 P_{nm}(r,\vartheta)P_{kl}(r,\vartheta)r\,\mathrm{d}r\,\mathrm{d}\vartheta = \delta_{nk}\delta_{ml}, \tag{2}$$

where $\delta_{nk}\delta_{ml}$ are Kronecker symbols and $r=1$ is the maximum scale of the object in the concrete scene. The radial function $J_n(r)$ and the Fourier angular kernel $\exp(\mathrm{j}m\theta)$ are separable. The $\exp(\mathrm{j}m\theta)$ is orthogonal and the radial function $J_n(\alpha,\beta,r)$ should be orthogonal in the interval $0 \leqslant r \leqslant 1$ too:

$$\int_0^1 J_n(r,\alpha,\beta)J_k(r,\alpha,\beta)r\,\mathrm{d}r = \delta_{nk}(\alpha,\beta). \tag{3}$$

In the orthogonal polynomial theory, the Jacobi polynomial $G_n(\alpha,\beta,r)$ is defined [6] as

$$G_n(\alpha,\beta,r) = \frac{n!(\beta-1)!}{(\alpha+n-1)!}\sum_{s=0}^{n}(-1)^s \times \frac{(\alpha+n+s-1)!}{(n-s)!s!(\beta+s-1)!}r^s. \tag{4}$$

Jacobi polynomial $G_n(\alpha,\beta,r)$ is orthogonal in the interval $0 \leqslant r \leqslant 1$:

$$\int_0^1 G_n(\alpha,\beta,r)G_m(\alpha,\beta,r)w(\alpha,\beta,r)\,\mathrm{d}r = b_n(\alpha,\beta)\delta_{nm}, \tag{5}$$

where $w(\alpha,\beta,r)$ is the weight function and b_n is the normalization constant:

$$b_n = \frac{n![(\beta-1)!]^2(\alpha-\beta+n)!}{(\beta+n-1)!(\alpha+n-1)!(\alpha+2n)}, \tag{6}$$

which are a function of the parameters α and β, and the general weight function:

$$w(\alpha,\beta,r) = (1-r)^{\alpha-\beta}r^{\beta-1} \quad \alpha-\beta > -1,\ \ \beta > 0. \tag{7}$$

In the above formulas, α and β are real parameters, the value variation of which will form different Jacobi polynomials. Comparing formulas (3) and (5), we can get the radial function set:

$$J_n(\alpha,\beta,r) = \sqrt{\frac{w(\alpha,\beta,r)}{b(\alpha,\beta)r}}\,G_n(\alpha,\beta,r). \tag{8}$$

In the polar coordinate system, an image function $f(r,\vartheta)$ can be decomposed into the superposition of weighted orthogonal components:

$$f(r,\vartheta) = \sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty}\phi_{nm}J_n(r)\exp(\mathrm{j}m\vartheta), \tag{9}$$