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Image analysis by circularly semi-orthogonal moments

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ABSTRACT

Various types of circularly orthogonal moments have been widely used for image reconstruction and rotation invariant classification. However, they suffer from two errors namely numerical integration error and geometric error, which affect their reconstruction capability and pattern recognition accuracy. In this paper, a novel category of circular moments named circularly semi-orthogonal moments is proposed. In the proposed moment, a set of orthogonal basis functions modulated by a negative power exponential envelope is utilized as the radial basis function. For a given degree n, the radial basis function possesses more compact bandwidth, less cutoff frequency and more zeros compared with the frequently-used circularly orthogonal moments including Zernike and orthogonal Fourier–Mellin moments, and so the circularly semi-orthogonal moment calculated with the zeroth order approximation is more robust to numerical error than the frequently-used circularly orthogonal moments. Furthermore, the capability of the semi-orthogonal moment to describe high spatial frequency components of images is relatively higher than that of the frequently-used circularly orthogonal moments. Experimental results demonstrate that the semi-orthogonal moments calculated with the zeroth order approximation perform better than the frequently-used circularly orthogonal moments in terms of image reconstruction capability and invariant recognition accuracy in noise-free, noisy and smooth distortion conditions. It is also shown that the proposed high order moments are more numerically stable than the circularly orthogonal moments.

1. Introduction

Description of images invariant to affine transformations such as translation, scaling and rotation is useful in image analysis, object recognition and classification [1]. Moments, as a popular class of the global invariant descriptors, have been widely used in image analysis, pattern recognition and computer vision applications [2–7].

The existing moments can be roughly divided into two categories: nonorthogonal moments and orthogonal moments. Nonorthogonal moments such as geometric moments [8] and complex moments [9] are components of the projection of an image onto a set of monomial functions. The orthogonal moments including the circularly orthogonal moments and Cartesian orthogonal moments are the projections of the image onto a set of orthogonal basis functions. The Cartesian orthogonal moments such as Legendre moment [10], discrete Tchebichef moment [11], Krawtchouk moment [12], dual Hahn moment [13] and Racah moment [14] have been defined in the Cartesian coordinates, where moment invariants particularly rotation invariants are not readily available. The circularly orthogonal moments including Zernike moment (ZM) [15] and orthogonal Fourier–Mellin moment (OFM) [16] are defined in the polar coordinates, their magnitudes are natively rotation invariant, and so have been widely used in many image processing, pattern recognition and computer vision applications [17–22]. In our early work [23], a kind of circularly orthogonal moment namely Bessel–Fourier moment (BFM) was proposed, and has been used in image reconstruction and recognition [24].

Despite the aforementioned important characteristic of the circularly orthogonal moments which is useful for image description, they suffer from two errors namely numerical integration error and geometric error, which affect their accuracy, thereby, affecting their rotational invariance and robustness to noise [25]. These errors depress their reconstruction accuracy and pattern recognition performance, and heavily degrade their extension applications as well. Furthermore, these errors lead to the high order moments numerically instable, which results in the failure of image reconstruction and classification with the high order moments, e.g., when the order of Zernike moments is increased up to 45, the numerical instability will lead to the complete failure of image reconstruction [26].

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Recently, some works [25–30] focused on improving the reconstruction accuracy and pattern recognition performance of the circularly orthogonal moments by alleviating the two errors. They alleviate the numerical integration error by increasing the sampling points [27] or performing exact integration of the kernel function in each square grid [25,26,28–30], and the geometric error is reduced by using different numerical integrations for three types of pixel grids [25]. Consequently, the reconstruction accuracy and pattern recognition performance of these methods have been improved to different degrees, but they are computationally expensive, which limits their extension applications. Although the integrations of the kernel functions in square grids can be pre-computed and stored for any future use [28], but they depend on the size of images, it is unpractical to pre-compute and store the integrations of the kernel functions for all potential image sizes. As a result, alternative circular moment that is robust to the numerical integration error with the zeroth order approximation is desired for computer vision applications especially for image pattern classification.

Besides the two errors, the overflow and finite precision errors should be considered in the calculations of the existing circularly orthogonal moments due to the fact that their radial basis functions include many factorial and power items. These factorial and power calculations not only represent a very significant part of the overall computation procedure but also influence the procedure with numerical instabilities especially for high-order moments. Papakostas et al. [31] improved the computational efficiency and numerical stability of orthogonal Fourier–Mellin moments via calculating the factorial and power items with recursive computation. Walia et al. calculated Zernike [32] and orthogonal Fourier–Mellin moments [33] by directly using their radial polynomials' recursive relations which are free from factorial and power terms. Hosny et al. [34] combined Papakostas's method with Xin's method [30] to calculate orthogonal Fourier–Mellin moments, and the numerical integration, overflow and finite precision errors were alleviated simultaneously.

This paper proposes a novel category of circular moments named as circularly semi-orthogonal moments based on a set of orthogonal basis functions modulated by a negative power exponential envelope, which radial basis function only involves several simple calculations and is free from factorial terms. Furthermore, its bandwidth is more compact and cutoff frequency is less than these of the orthogonal basis functions, and so the proposed semi-orthogonal moment is free of the overflow and finite precision errors and more robust to numerical error than the frequently-used circularly orthogonal moments. Experimental results show the superiority of this moment compared with the circularly orthogonal moments in terms of image reconstruction capability and invariant recognition accuracy. The outline of this paper is as follows: Section 2 will provide the definition, calculation and property of the circularly semi-orthogonal

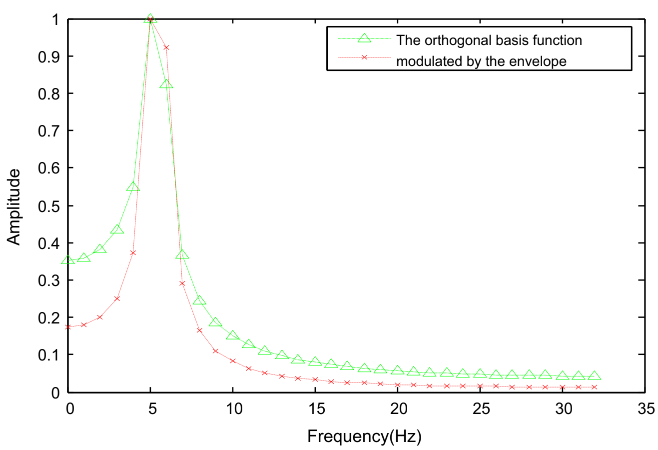

Fig. 1. The frequency characteristics of the orthogonal basis function ($\sin(n+1)\pi r/\sqrt{r/2}$) with $n=10$ and its modulation version with the envelope.

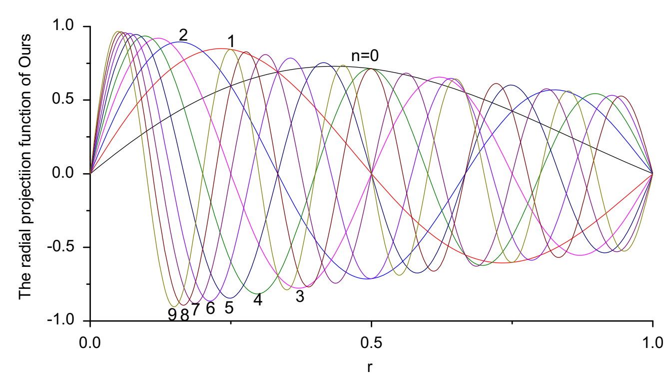

Fig. 3. The plot of the radial polynomial of the semi-orthogonal moment with $n=0,\cdots,9$.

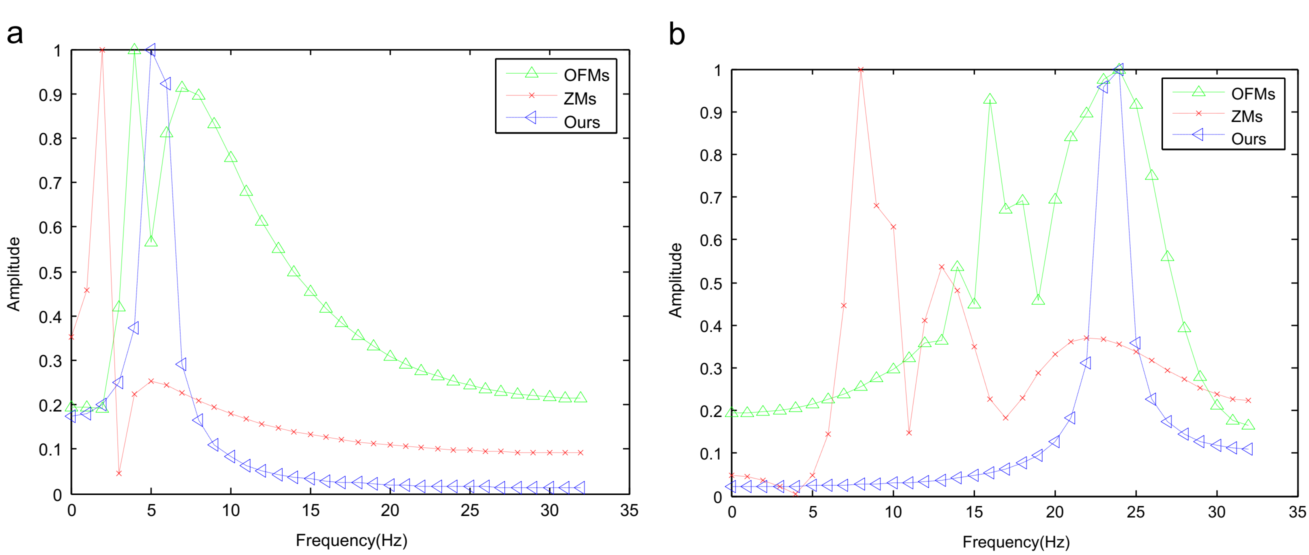

Fig. 2. The frequency characteristics of Zernike, orthogonal Fourier–Mellin and our moments' radial polynomials with (a) $n=10$; (b) $n=46$.