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$(2D)^2$LDA: An efficient approach for face recognition

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Abstract

Although 2DLDA algorithm obtains higher recognition accuracy, a vital unresolved problem of 2DLDA is that it needs huge feature matrix for the task of face recognition. To overcome this problem, this paper presents an efficient approach for face image feature extraction, namely, $(2D)^2$LDA method. Experimental results on ORL and Yale database show that the proposed method obtains good recognition accuracy despite having less number of coefficients.

Keywords: Principal component analysis; Linear discriminant analysis; Face recognition

1. Introduction

Linear discriminant analysis (LDA) is a well-known feature extraction and data representation technique widely used in the areas of pattern recognition for feature extraction and dimension reduction. The objective of LDA is to find the optimal projection so that the ratio of the determinants of the between-class and the within-class scatter matrices of the projected samples reaches its maximum. However, concatenating 2D matrices into 1D vectors leads to very high dimensional nature of image vector, where it is difficult to evaluate the scatter matrices accurately due to its large size and the relatively small number of training samples. Furthermore, the within-class scatter matrix is always singular, making the direct implementation of LDA algorithm an intractable task.

To overcome these problems, a new technique called 2DLDA [1] was recently proposed, which directly computes eigenvectors of the so called scatter matrices without matrix-to-vector conversion. Because the size of the scatter matrices is equal to the width of the images, which is quite small compared to the size of the scatter matrices in LDA, 2DLDA evaluates the scatter matrices more accurately and computes the corresponding eigen vectors more efficiently. It was reported in Ref. [1] that the recognition accuracy on several databases was higher using 2DLDA than other PCA and LDA-based algorithms.

However, the main drawback of 2DLDA is that it needs more coefficients for image representation than conventional PCA- and LDA-based schemes. For an image size of 112×92, the commonly used image size in face recognition, the number of coefficients used by 2DLDA for classification is $112 \times d$, where d is set to no less than 5 for satisfactory accuracy.

In this paper, we first indicate that 2DLDA is essentially working in the row-direction of images, and then propose an alternative 2DLDA which works in the column direction of images. By simultaneously combining row and column directions, we develop two-directional 2DLDA, i.e. $(2D)^2$LDA, for efficient representation and recognition. Experimental results on ORL and Yale database shows that the proposed method obtains same or even better recognition accuracy than 2DLDA, while the number of coefficients needed by the former for image representation is much smaller than that of the latter.

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2. Overview of 2DLDA approach

2DLDA is an effective feature extraction and discrimination approach [1] in face recognition. Formally, it can briefly be formulated as follows: Suppose $\{A_k\}_{k=1}^N$ are the training images, which contain C classes, and the ith class C_i has n_i samples ($\sum_{i=1}^C n_i = N$). 2DLDA attempts to seek a set of optimal discriminating vectors to form a transform $X_d = \{x_1, x_2, \ldots, x_{d\}}$ by maximizing the 2D Fisher criterion denoted as

$$J(X) = \frac{X^T G_b X}{X^T G_w X}. \tag{1}$$

In Eq. (1), T denotes matrix transpose, G_b and G_w, respectively, are between-class and within-class scatter matrices:

$$G_b = \frac{1}{N}\sum_{i=1}^{C} n_i(\overline{A}_i - \overline{A})^T(\overline{A}_i - \overline{A}), \tag{2}$$

$$G_w = \frac{1}{N}\sum_{i=1}^{C}\sum_{j\in C_i}(A_j - \overline{A}_i)^T(A_j - \overline{A}_i), \tag{3}$$

$\overline{A}_i, \overline{A}$ denote the means of ith class and the whole training set, respectively. A_j is the jth image in the class C_i. The goal of 2DLDA scheme is to find the optimal discriminating vectors X_{opt} in order to maximize $J(X)$. Obviously, the optimal discrimination vectors X_{opt} are the eigenvector corresponding to the dominant eigenvalues of eigenstructure $G_w^{-1}G_b$. It has been proved that the optimal value for the discriminating vectors X_{opt} is composed of the orthonormal eigenvectors $x_1, x_2, \ldots, x_d$ of $G_w^{-1}G_b$ corresponding to the d largest eigenvalues. Now, given an image $A_{m\times n}$, all the projections of the image matrix in the d-directions make up md-dimensional vector, which is the 2DLDA feature vector.

2.1. Proposed alternative-2DLDA

Let $A_k = [(A_k^{(1)})^T, (A_k^{(2)})^T, \ldots, (A_k^{(m)})^T]^T$, $\overline{A}_i = [(\overline{A}_i^{(1)})^T, (\overline{A}_i^{(2)})^T, \ldots, (\overline{A}_i^{(m)})^T]^T$, $\overline{A} = [(\overline{A}^{(1)})^T, (\overline{A}^{(2)})^T, \ldots, (\overline{A}^{(m)})^T]^T$, where $A_k^{(j)}, \overline{A}_i^{(j)}, \overline{A}^{(j)}$ denote the jth row vectors of A_k, $\overline{A}_i$ and $\overline{A}$, respectively. Then Eqs. (2) and (3) can be written as:

$$G_b = \frac{1}{N}\sum_{i=1}^{C} n_i \sum_{j=1}^{m}(\overline{A}_i^{(j)} - \overline{A}^{(j)})^T(\overline{A}_i^{(j)} - \overline{A}^{(j)}), \tag{4}$$

$$G_w = \frac{1}{N}\sum_{i=1}^{C}\sum_{k\in C_i}\sum_{j=1}^{m}(A_k^{(j)} - \overline{A}_k^{(j)})^T(A_k^{(j)} - \overline{A}_k^{(j)}). \tag{5}$$

Eq. (5) reveals that the scatter matrix G_w can be obtained from the outer product of row vectors of images, assuming the training images have zero mean [2]. For this reason, we claim that original 2DLDA is working in the row direction of images. Apparently, a natural extension is to use the outer product between column vectors of images to construct G_b and G_w.

Let

$$A_k = [(A_k^{(1)}), (A_k^{(2)}), \ldots, (A_k^{(n)})],$$
$$\overline{A}_i = [(\overline{A}_i^{(1)}), (\overline{A}_i^{(2)}), \ldots, (\overline{A}_i^{(n)})],$$
$$\overline{A} = [(\overline{A}^{(1)}), (\overline{A}^{(2)}), \ldots, (\overline{A}^{(n)})],$$

where $A_k^{(j)}, \overline{A}_i^{(j)}, \overline{A}^{(j)}$, respectively denote the jth column vectors of A_k, $\overline{A}_i$ and$\overline{A}$.

Let Z denotes an m-dimensional unitary column vector. Projecting the image matrix $A_{m\times n}$ onto Z yields a $q \times n$ feature matrix, i.e, $B = Z^T A$. Similar to Eq. (1), the following criterion is adopted to find the optimal projection vector Z and is given by $J(Z) = trace(S_b^z)/trace(S_w^z)$, where S_b^z and S_w^z are, respectively, given by $1/N\sum_{i=1}^C n_i(\overline{y}^i - \overline{y})(\overline{y}^i - \overline{y})^T$ and $1/N\sum_{i=1}^C\sum_{j\in C_i}(y_j - \overline{y}^i)(y_j - \overline{y}^i)^T$. Here $\overline{y}$ and $\overline{y}^i$, respectively, denote the global and the mean vector of ith class in the projection space.

It is easy to verify that $trace(S_b^z) = Z \cdot G \cdot Z^T$ and $trace(S_w^z) = Z \cdot G_w \cdot Z^T$ where G_b and G_w are now given as

$$G_b = \frac{1}{N}\sum_{i=1}^{C} n_i \sum_{j=1}^{m}(\overline{A}_i^{(j)} - \overline{A}^{(j)})(\overline{A}_i^{(j)} - \overline{A}^{(j)})^T, \tag{6}$$

$$G_w = \frac{1}{N}\sum_{i=1}^{C}\sum_{k\in C_i}\sum_{j=1}^{m}(A_k^{(j)} - \overline{A}_k^{(j)})(A_k^{(j)} - \overline{A}_k^{(j)})^T. \tag{7}$$

Similarly, the optimal projection matrix $Z_{\text{opt}} = [z_1, z_2, \ldots, z_q]$ can be obtained by computing the orthonormal eigenvectors of $G_w^{-1}G_b$ corresponding to the q largest eigenvalues thereby maximizing $J(Z)$.

2.2. Proposed (2D)^2 LDA method: 2-directional 2-dimensional LDA

We reasoned in Section 2.1 that 2DLDA works in the rowwise direction reflecting the information between row of images to learn an optimal matrix X from a set of training images, and then project an $m \times n$ image A onto X, yielding m by d matrix, i.e. $Y_{m\times d} = A_{m\times n} \cdot X_{n\times d}$. Similarly, the alternative 2DLDA learns optimal projection matrix Z reflecting information between columns of images and then projects A onto Z, yielding a q by n matrix, i.e. $B_{q\times n} = Z_{m\times q}^T \cdot A_{m\times n}$.

Suppose we have obtained the projection matrices X (as in Section 2) and Z (as in Section 2.1), projecting the m by n image A onto X and Z simultaneously, yielding a q by d matrix C,

$$C = Z^T \cdot A \cdot X. \tag{8}$$