Pattern Recognition 48 (2015) 1391–1403

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Estimation of linear deformations of 2D and 3D fuzzy objects

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ARTICLE INFO

Article history:
Received 15 March 2012
Received in revised form
22 September 2014
Accepted 2 October 2014
Available online 14 October 2014

Keywords:
Registration
Affine transformation
System of polynomial equations
Fuzzy representation

ABSTRACT

Registration is a fundamental task in image processing, it is used to determine geometric correspondences between images taken at different times and/or from different viewpoints. Here we propose a general framework in *n*-dimensions to solve binary shape/object matching problems without the need of establishing additional point or other type of correspondences. The approach is based on generating and solving polynomial systems of equations. We also propose an extension which, provided that a suitable segmentation method can produce a fuzzy border representation, further increases the registration precision. Via numerous synthetic and real test we examine the different solution techniques of the polynomial systems of equations. We take into account a direct analytical, an iterative least-squares, and a combined method. Iterative and combined approaches produce the most precise results. Comparison is made against competing methods for rigid-body problems. Our method is orders of magnitude faster and is able to recover alignment regardless of the magnitude of the deformation compared to the narrow capture range of others. The applicability of the proposed methods is demonstrated on real X-ray images of hip replacement implants and 3D CT volumes of the pelvic area. Since the images must be parsed through only once, our approach is especially suitable for solving registration problems of large images.

1. Introduction and previous work

Image registration is one of the main tasks of image processing, its goal is to find the geometric correspondence between images. If image segmentation is available such that objects are defined, registration is commonly performed by shape matching. First, an arbitrary *segmentation step* provides the *shapes* and then the shapes are registered.[1] This solution is especially viable when the image intensities undergo strong nonlinear deformations that are hard to model, *e.g.* in case of X-ray imaging. If there are clearly defined regions in the images (*e.g.* bones or implants in X-ray images), a rather straightforward segmentation method can be used to define their shapes adequately. 3D imaging in medical and industrial applications is common nowadays. Binary or fuzzy 3D objects might be produced by the imaging process (*e.g.* discrete tomography), generated from geometric descriptions (*e.g.* from CAD models), or by segmenting corresponding regions from non-binary images.

Classical methods solve the registration problem by either extracting *geometric features* or using the *image intensities* directly, and try to establish correspondences by usually applying an iterative technique [1]. Geometric features can be *e.g.* interest points with descriptors [2,3], surfaces [4], skeletons [5] or more general sets of points [6]. Intensity similarity methods are used mainly for non-binary single- and multi-modality medical registration problems, but can also be applied for registration of binary images. An important decision is the type of geometric transformation to consider. Non-linear transformations can be used to *e.g.* model tissue movements and local changes of shapes. These algorithms must take into account the deformation parameters of different tissue types which can be a hard, application specific and time consuming task. However, in many scenarios global linear transformations are sufficient, especially if fast registration is necessary.

A parametric estimation method was proposed by Hagege et al. [7,8] for grayscale and color images. By using intensity information, the result is provided solving a *linear* system of equations. Domokos et al. proposed a modification [9,10] to this approach to deal with affine matching of binary shapes solving a *polynomial* system of equations. In previous papers a variant of this approach was compared against other recent 2D shape registration methods [11,12]. An alternative approach is to use covariant Gaussian

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[1] Later on we use the term 'object' interchangeably with 'shape'.

http://dx.doi.org/10.1016/j.patcog.2014.10.006
0031-3203/

densities to produce a linear system of equations to solve the matching problem [13].

In [14], Ma et al. propose an approach for registration of serial sections of microscope images. After a binarization step, an initial rigid transformation is computed based on first (translation) and higher order moments (rotation) of the binary objects. Then a correlation-based iterative technique is applied that optimizes the overlap between the objects. It takes around 6 s to register one 2D section pair using the method of Ma et al. The method presented in this paper has some similarities with the method of Ma et al. but is significantly faster. Mai et al. proposed a fast approach in [15] for matching of closed 2D and 3D curves by minimizing a subspace projection error, however an extension to 3D objects is not available.

Fast rigid-body registration of bone structures is important in image guided surgical planning and execution for registering pre-operative volumes to intra-operative ones. Zhang et al. give an overview of surface based registration techniques and propose a 15 times faster method than standard Iterative Closest Point (ICP) methods [4]. However, it still takes around 1 min to register object models segmented from high resolution CT images. In [16], hybrid dimensionality-reduction shape descriptors are introduced to rigidly register 3D shapes represented as surface meshes. A direct method of Burel et al. uses spherical harmonics to recover the orientational differences between surfaces of 3D objects [17]. It provides fast registration for a *rigid-body* case. Rigid registration of thoracic images is also applicable to *e.g.* detections of lymphoma and changes over time using PET-CT scanners. PET images delineate the uptake of the contrast agent in organs (lymph nodes), while the CT modality can be used for registration and morphological localization. Here non-rigid registrations are discouraged since these could change the size of the organs. Automatic initial placement of organ models can also benefit from fast linear registrations [18].

In this paper we propose a unified framework for solving registration problems of n-dimensional binary objects. There is no need of establishing additional point or other type of correspondences. The approach is based on generating and solving polynomial systems of equations. The system is generated by a single pass over the image. The solution does not depend on the size of the objects. This makes our method especially suitable for registering large volume images.

In Section 2, we describe our proposed method in general n-dimensional continuous space, based on our previous papers related to 2D and 3D problems [9,10,19]. Since digital images are discrete, continuous case solutions must be adjusted to that; an approximative formula by discretization of the space is derived in Section 3. Nowadays, image processing and analysis methods based on fuzzy sets and fuzzy techniques are attracting increasing attention. Preserving fuzziness in the image segmentation, and thereby postponing decisions related to crisp object definitions has many benefits, such as reduced sensitivity to noise, improved robustness and increased precision of feature measures. We incorporate the use of fuzzy object representation based on pixel/voxel coverages into our registration method.

The major new contribution of this paper is the detailed discussion and evaluation of the fast and efficient algorithmic solution techniques of the polynomial system of equations. We also explain why the method in [10] produced unresolved cases and sub-optimal results, and show that by applying more sophisticated solution selection techniques, those cases can be completely eliminated and the precision can be significantly improved. In Section 4, we propose an iterative solution of overdetermined systems, a direct analytical solution of non-singular systems, and a combined method exploiting the benefits of the former two. Extensive synthetic tests show the effect of fuzzy representation, the performance of the different solution techniques, and reveal the robustness of our method against different types of image degradations in Section 5. In previous papers we gave only few examples of real registration problems. Here we demonstrate the usability of our method based on 18 image pairs of 2D X-ray of bone implants, and 18 pairs of 3D pelvic CT studies (Section 6).

2. Parametric estimation of affine deformations

We briefly overview the affine registration approach from [10]. The formulation is extended here to any dimension n. Let us denote the points of the *template* and the *observation* by $\mathbf{x}, \mathbf{y} \in \mathbb{P}^n$, respectively, in the projective space. The projective space allows simple notation for affine transforms and assumes using of homogeneous coordinates. Since affine transformations never alter the last (homogeneous) coordinate of a point, which is always equal to 1, we, for simplicity, and without loss of generality, liberally interchange between projective and Euclidean space, keeping the simplest notation.

Let $\mathbf{A}$ denote the unknown n-dimensional affine transformation that we want to recover. We can define the identity relation as follows:

$$\mathbf{A}\mathbf{x} = \mathbf{y} \Leftrightarrow \mathbf{x} = \mathbf{A}^{-1}\mathbf{y}.$$

The above equations still hold when a properly chosen function $\omega : \mathbb{P}^n \to \mathbb{P}^n$ is acting on both sides of the equations [9]

$$\omega(\mathbf{A}\mathbf{x}) = \omega(\mathbf{y}) \Leftrightarrow \omega(\mathbf{x}) = \omega(\mathbf{A}^{-1}\mathbf{y}). \tag{1}$$

Binary images do not contain radiometric information, therefore they can be represented by their characteristic function $\mathbb{1} : \mathbb{P}^n \to \{0, 1\}$, where 0 and 1 are assigned to the elements of the background and foreground, respectively. Let $\mathbb{1}_t$ and $\mathbb{1}_o$ denote the characteristic function of the *template* and *observation*, respectively. In order to avoid the need for point correspondences, we integrate over the foreground domains $\mathcal{F}_t = \{\mathbf{x} \in \mathbb{P}^n | \mathbb{1}_t(\mathbf{x}) = 1\}$ and $\mathcal{F}_o = \{\mathbf{y} \in \mathbb{P}^n | \mathbb{1}_o(\mathbf{y}) = 1\}$ of the *template* and the *observation*, respectively, yielding [9]

$$|\mathbf{A}| \int_{\mathcal{F}_t} \omega(\mathbf{x})\, d\mathbf{x} = \int_{\mathcal{F}_o} \omega(\mathbf{A}^{-1}\mathbf{y})\, d\mathbf{y} \quad \text{and} \tag{2}$$

$$\int_{\mathcal{F}_t} \omega(\mathbf{A}\mathbf{x})\, d\mathbf{x} = \frac{1}{|\mathbf{A}|} \int_{\mathcal{F}_o} \omega(\mathbf{y})\, d\mathbf{y}, \tag{3}$$

where $d\mathbf{x}$ is the n-dimensional volume differential. The Jacobian of the transformation ($|\mathbf{A}|$) can be easily evaluated as

$$|\mathbf{A}| = \frac{\int_{\mathcal{F}_o} d\mathbf{y}}{\int_{\mathcal{F}_t} d\mathbf{x}}. \tag{4}$$

The basic idea of the proposed approach is to generate sufficiently many linearly independent equations by making use of the relations in Eqs. (1)–(3). We need at least as many equations as the degrees of freedom (DOF) of the n-dimensional affine transformation. It is 6 and 12 in 2D and 3D, respectively. We can select the ω-functions almost arbitrarily (polynomials, trigonometric functions, exponentials, etc.). From a computational point of view, especially for a direct analytical solution, we need a system which can be solved efficiently in practice. For that, we have to carefully select the generator functions ω. Of course, a linear system of equations is the simplest and most efficient to solve analytically, however there are no such ω functions to generate sufficiently many independent linear equations to solve the system. Therefore nonlinear ω functions have to be used, and since they are applied to the unknown parameters, the system becomes nonlinear. In particular, the direct solution of these systems usually requires the construction of a Gröbner basis,