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Representation of enclosing surfaces from simple voxelized objects by means of a chain code

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ABSTRACT

A chain code for representing three-dimensional (3D) simple objects is defined. Once digitalized, any solid composed of voxels and homeomorphic to the sphere can be described by means of a codified sequence of faces in the enclosing surface. This sequence is obtained from a Hamiltonian cycle in the *face adjacency graph* of such a surface. For the proposed code each chain element takes one of nine possible values and the length of a chain is determined by the number of faces in the surface. Since this code only considers relative changes of direction, the descriptor is invariant under rotation and translation. We also show some simple operations over the chain to make this descriptor invariant under mirroring and complement transformations. Finally, we present some results of this code applied to large objects and demonstrate its convenience over other codes. Part of the relevance of this work is the lossless compact representation of 3D objects by using a single chain regardless of its position and orientation.

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1. Introduction

The representation of 3D objects is an important topic in pattern recognition and computer vision. This work deals with the analysis and description of simple voxelized objects homeomorphic to the sphere. For a binary solid, its enclosing surface is obtained and then described by means of codifying a Hamiltonian cycle in such a surface. Chain code methods are widely used because they preserve information and allow considerable data reduction. Additionally, chains use a finite alphabet and allow the use of grammatical techniques for object analysis and morphological transformations.

One of the first approaches to represent surfaces using a descriptor was presented by Ansaldi et al. [1]. That work proposes the use of a graph known as the *face adjacency graph* in which its nodes represent the faces of the enclosing surface, whereas two nodes of the graph are connected by an arc if their respective faces are adjacent by an edge.

Many problems in computer science may be reduced to the problem of determining whether a graph contains a Hamiltonian path and there have been several reported results to solve this problem under particular conditions [2]. In this work we take advantage of one of these results to find Hamiltonian cycles in 4-connected planar graphs as a way to represent the enclosing surface of 3D objects.

Tutte [3,4] demonstrated that every 4-connected planar graph has a Hamiltonian cycle. Later, Thomassen [5] extended Tutte's result proving that not only do these graphs contain a Hamiltonian cycle but also a Hamiltonian path between every pair of their nodes. Based on this proof, Chiba and Nishizeki [6] presented an algorithm to find such cycles in linear time complexity.

A direct consequence of these results is that the problem of describing a 3D object can be seen as one of finding a Hamiltonian cycle within its surface. That is, once the face adjacency graph of a solid is obtained, we may have a one-dimensional descriptor by means of finding and codifying a Hamiltonian cycle in such a graph. There have been several methods proposed to codify discrete curves [7–10]. In this paper, we introduce a new chain code to represent any sequence of oriented faces which are progressively connected by an edge. This new code is, to the best of our knowledge, the first one used to represent surfaces directly. Our code uses nine relative changes of direction and it is invariant under translation and rotation. Optionally, it may be also invariant under mirroring and complement transformations.

This paper is organized as follows. Section 2 gives the basis to obtain a 4-connected graph for any binary solid and that is used to obtain the Hamiltonian cycle to be codified into a chain. In Section 3 we define the new chain code to represent surfaces. Then, in

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Section 4 we present some results of describing large voxelized volumes with the proposed code. Finally, in Section 5 we discuss our results and give some conclusions.

2. Hamiltonian cycle extraction

In this section, we explain how to obtain a sequence of all the faces in the enclosing surface of a voxelized solid. We may consider a voxelized solid as a discrete function $S : \mathbb{Z}^3 \rightarrow \{0, 1\}$ where the points representing the interior of the solid are assigned a value 1 and those outside the solid are assigned a value of 0. Some assumptions in this work are that (1) voxelized solids have been isolated (i.e., segmented) and (2) their enclosing surfaces were already extracted, for example, using a boundary tracking algorithm such as that proposed by Artzy [11]. It is worth mentioning some advantages of using this kind of surfaces over other polygonizations (e.g., Marching Cubes [12]). Besides the time and space efficiencies, algorithms for surface extraction produce surfaces that do not present some issues related to the over tessellation and geometrical ambiguities of meshes [13]. Additionally, this kind of surfaces can alternatively be represented by a face adjacency graph.

Before going any further, we introduce some concepts of graph theory used in this paper [14]. Firstly, we define a *graph* as the ordered pair $G = (V, E)$ of a set V of nodes and a set E of arcs such that $E \subseteq V^2$. A *planar graph* is a graph that can be embedded in a plane in such a way that no arcs cross each other. In this context, a *path* P is a non-empty graph of the form $V = \{x_0, x_1, \ldots, x_k\}$ and $E = \{x_0x_1, x_1x_2, \ldots, x_{k-1}, x_k\}$ and we say that P is a *simple path* if all its nodes are distinct. For $k \geq 3$ the graph $C = P + x_{k-1}x_0$ is called a *cycle*. For our work it is important to notice that if a simple path in G contains every node of the graph, then it is called a *Hamiltonian path*. Similarly, we define a *Hamiltonian cycle* for a cycle in G. We also say that a non-empty graph G is *connected* when there is a path between every pair of its nodes. Additionally, a graph G is said to be *k-connected* if it remains connected after removing any subset of $k-1$ nodes with their connecting arcs.

As we mentioned before, face adjacency graphs are used to represent surface faces and their adjacency. In Fig. 1 we show an example of a face adjacency graph for a simple voxelized solid; Fig. 1(a) shows the enclosing surface of a discrete solid composed of six voxels and Fig. 1(b) presents the face adjacency graph of that surface. Iin Fig. 1(c) the enclosing surface is superimposed with its adjacency graph.

In this paper we consider that voxels of an object are either connected by face or by edge. So, we exclude voxels connected by vertex. It is easy to see that for this kind of solids their adjacency graphs will be 4-connected and, moreover, those graphs will be planar provided that the solid contains no holes. Let us note planarity as a characteristic owned by graphs representing objects with no holes (i.e., homeomorphic to the sphere). In these graphs a Hamiltonian cycle represents that sequence of unrepeated faces, progressively connected by edge and whose first and last faces are also connected by edge. For 4-connected planar graphs it has been proved that such a cycle always exists [4,5,15]. In Fig. 1(b) we outline, using thicker lines, a Hamiltonian cycle while in Fig. 1(d) we present the same cycle superimposed over its surface.

Regarding the adjacency graph of objects with holes, there exist formal conjectures stating their Hamiltonicity [16,17]. In particular, the main conjecture in [18] states that the enclosing surface of any solid composed of voxels is Hamiltonian. Despite these last results, in this paper we bound our contribution to the representation of objects with no holes given the absence of suitable methods to find cycles in polynomial-time complexity for non-planar graphs.

In general, the Hamiltonian path problem is NP-complete; however, when it comes to 4-connected planar graphs the problem becomes computable in polynomial-time complexity [6,19], something that means a feasible solution to the problem

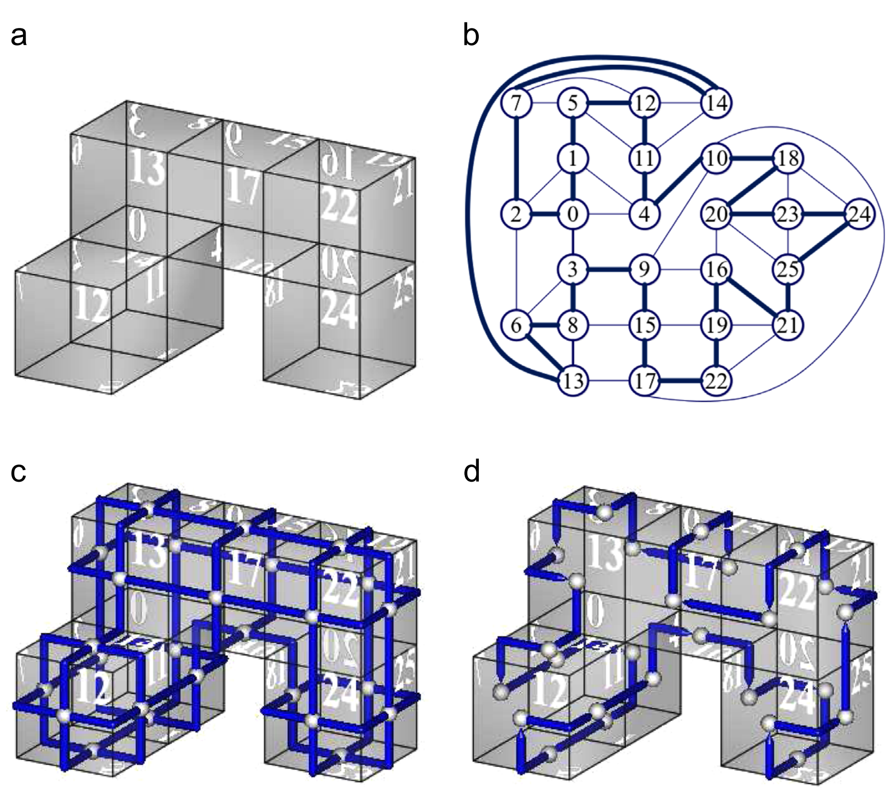

Fig. 1. (a) The enclosing surface of a voxelized solid composed of 26 labeled faces; (b) the face adjacency graph of the surface in (a) outlining a Hamiltonian cycle with thicker lines; (c) the enclosing surface in (a) with its face adjacency graph superimposed; (d) the enclosing surface in (a) with the Hamiltonian cycle in (b) superimposed.

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