Pattern Recognition 47 (2014) 1751–1763

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

A general non-local denoising model using multi-kernel-induced measures

Zhonggui Sun [a,b], Songcan Chen [a,*], Lishan Qiao [b]

[a] College of Computer Science and Technology, Nanjing University of Aeronautics & Astronautics, 210016 Nanjing, PR China
[b] Department of Mathematics Science, Liaocheng University, 252000 Liaocheng, PR China

ARTICLE INFO

Article history:
Received 2 August 2012
Received in revised form
26 October 2013
Accepted 4 November 2013
Available online 15 November 2013

Keywords:
Image denoising
Multi-kernel learning
Kernel-induced measure
Non-local means (NLM)
Complicated noise

ABSTRACT

Noises are inevitably introduced in digital image acquisition processes, and thus image denoising is still a hot research problem. Different from local methods operating on local regions of images, the non-local methods utilize non-local information (even the whole image) to accomplish image denoising. Due to their superior performance, the non-local methods have recently drawn more and more attention in the image denoising community. However, these methods generally do not work well in handling complicated noises with different levels and types. Inspired by the fact in machine learning field that multi-kernel methods are more robust and effective in tackling complex problems than single-kernel ones, we establish a general non-local denoising model based on multi-kernel-induced measures (GNLMKIM for short), which provides us a platform to analyze some existing and design new filters. With the help of GNLMKIM, we reinterpret two well-known non-local filters in the united view and extend them to their novel multi-kernel counterparts. The comprehensive experiments indicate that these novel filters achieve encouraging denoising results in both visual effect and PSNR index.

1. Introduction

Denoising is still a very active area of research in image processing, which goal is to reduce noise artifacts while retaining good details (such as edges) of observed images as much as possible [1]. Towards the end, many denoising methods, such as the mean filter, the total variation filter [2], the bilateral filter [3] and so on, were sequentially proposed and have obtained their popularity. Their common and effective characteristic is that they were all realized by locally averaging [4] which is an operation on a set of image values limited to a local targeted region in image. Though able to preserve image details to different degrees, the aforementioned local methods have two weaknesses: (1) the weights involved in the averaging only depend on the single pixels, thus sensitive to the polluted pixels; (2) artificial shocks (unpredictable artificial stripes or textures) usually appear in the denoised results [4].

To mitigate the weaknesses of local methods, Buades et al. proposed a non-local means filter (NLM) recently [4]. Unlike the local filters which typically operate on a *local* neighborhood, NLM operates on a *non-local* area (even the whole image) by using a dissimilarity measure between patches. Despite its simplicity and intuition in idea, NLM has been empirically validated to clearly outperform other classic filters including the aforementioned ones [5]. Inspired by such a patch-based and non-local viewpoint, many state-of-the-art non-local filters, including block-matching and three-dimensional filter (BM3D) [6], K-SVD [7], have been proposed in the recent years (refer to [5,8] for surveys in this topic).

In practice, the noise in an image is generally complicated, which can belong to different levels (strength) and different types (single or mixed) [9,10]. Although non-local methods have shown excellent performances in denoising, when faced with such complicated noises, they cannot necessarily be guaranteed to yield desirable denoising effects [11]. Take NLM as an example, it cannot keep high effectiveness in removing Gaussian noises with high levels, as shown in our experiment sections later. Also, as pointed out in [10], the NLM fails to remove the common mixed noise either. Therefore, the denoising ability of NLM still needs to be improved.

In terms of Huber robust statistics [12], robust filters are insensitive to outliers and can preserve image details well. Inspired by [12], Tan et al. developed a set of robust measures with the kernel-induced distances and then invented a more general filtering model (KIM for short) in [13], which not only motivates new robust filters to born but also accommodates the typical filters including the mean filter and the median filter as its special cases. Meanwhile, as a powerful learning paradigm in the machine learning community, it has been proved that multi-kernel methods are more flexible in handling complicated learning tasks than single-kernel ones [14]. For our current image denoising

* Corresponding author. Tel.: +86 25 84892956.
E-mail address: s.chen@nuaa.edu.cn (S. Chen).

0031-3203/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.patcog.2013.11.003

problem, the noises in images often have different levels and different types. Thus we conjecture that a filter designed by replacing single-kernel with multi-kernels can be more powerful in removing such complicated noises.

Motivated by such successful factors in NLM, KIM and multi-kernel methods, we develop a novel denoising model in this paper, which is general, non-local, based on multi-kernel-induced measures and thus named as GNLMKIM for short.

The main contributions of this paper are listed as follows:

(1) We propose a new denoising model (GNLMKIM), which provides a general platform for analyzing and designing filters. Besides the multi-kernel strategies, the novel model combines several favorable properties, such as robustness and non-locality of current effective filters. For such a proposal, we are not aware of similar works in literature.
(2) To illustrate the generality and effectiveness of GNLMKIM, we focus on two popular non-local filters including the NLM [4] and MNF (mixed noise filter) [10] as the starting point of our work and show that these two filters are both special cases of GNLMKIM.
(3) Based on GNLMKIM, we further extend both NLM and MNF to their multi-kernel counterparts with encouraging experimental results in removing wide-level (no matter low or high) Gaussian noises and mixed noises respectively.

The rest of this paper is structured as follows: In Section 2, we briefly review related works. Section 3 describes and solves GNLMKIM in detail. In Sections 4 and 5, we first reinterpret NLM and MNF as special cases of GNLMKIM, and then extend them to their multi-kernel counterparts followed with encouraging experimental results. Finally, we conclude this paper in Section 6.

2. Related works

Some related works including noise models, NLM, MNF, KIM and kernel strategy will be briefly introduced in turn in this section.

2.1. Introduction to noise models

Here we concern three main noise models corresponding to Gaussian noise, impulse noise and their mixture respectively. The Gaussian noise model is mathematically defined as

$$Y = X + N \tag{1}$$

where Y is the observed (noisy) image, X is the noise-free (original) one to be recovered, N is the zero-mean Gaussian white noise with the standard deviation (STD) σ. A bounded domain on which X, Y, N are defined is denoted as $\Omega = [1, \ldots, m] \times [1, \ldots, n]$.

There are mainly two common types of impulse noises, i.e., salt-and-pepper noise and random-valued noise. Let N denote impulse noise. And suppose the gray values of an image are in the dynamic range $[N_{\min}, N_{\max}]$. For any pixel coordinate$i = (i_1, i_2) \in \Omega$, the two types of the impulse noises are respectively modeled as follows:

For the salt-and-pepper noise, we have

$$Y(i) = \begin{cases} N_{\min} & \text{with probabilty} \quad s/2, \\ N_{\max} & \text{with probabilty} \quad s/2, \\ X(i) & \text{with probabilty} \quad 1-s. \end{cases} \tag{2}$$

where $0 \le s \le 1$ is the salt-and-pepper noise level.

For the random-valued impulse noise,

$$Y(i) = \begin{cases} N(i) & \text{with probabilty} \quad r, \\ X(i) & \text{with probabilty} \quad 1-r. \end{cases} \tag{3}$$

where $0 \le r \le 1$ is the noise level. The gray values, $N(i)$s, are identically and uniformly distributed random numbers in $[N_{\min}, N_{\max}]$. Throughout this paper, unless specified, the impulse noise means the random-valued type.

Finally the mixed noise model can be formulated as

$$Y = N_{imp}(X + N) \tag{4}$$

where N_{imp} denotes an operator of image degradation with impulse noise.

2.2. Introduction to NLM

NLM is mainly designed for removing Gaussian noise. For any pixel coordinate $i = (i_1, i_2) \in \Omega$, NLM computes the restored (estimated) gray value $\hat{X}(i)$ as a weighted average in terms of

$$\hat{X}(i) = \frac{1}{C(i)} \sum_{j \in S_i} \exp\left(-\frac{||Y_i - Y_j||^2}{h^2} \right) Y(j) \tag{5}$$

where $S_i \subseteq \Omega$ is a non-local search window (even the whole Ω itself). Let $N^d\{i\} = \{(i_1 + j_1, i_2 + j_2) | (j_1, j_2) \in Z^2, |j_1| \le d, |j_2| \le d\}$. The restriction (patch) of Y to the neighborhood of coordinate i is defined as the vector $Y_i = Y(N^d\{i\}) = (Y(j) | j \in N^d\{i\})$. h^2 acts as a filtering parameter, which is recommended a value between 10σ and 15σ (σ is the noise STD) [4,5,15]. $C(i) = \sum_{j \in S_i} \exp(||Y_i - Y_j||^2_{2,\alpha}/h^2)$ is a normalized factor. $||Y_i - Y_j||^2_{2,\alpha}$ is a Gaussian weighted dissimilarity measure (distance) between Y_i and Y_j and defined as

$$||Y_i - Y_j||^2_{2,\alpha} = \sum_{k \in K} G_\alpha(k)(Y(i-k) - Y(j-k))^2 \tag{6}$$

where $K = \{(k_1, k_2) | |k_1| \le d, |k_2| \le d\}$ is a local neighborhood centered at the origin. And

$$G_\alpha(k) = \frac{1}{2\pi\alpha^2} \exp\left(-\frac{{k_1}^2 + {k_2}^2}{2\alpha^2} \right), \quad k = (k_1, k_2) \tag{7}$$

is the Gaussian kernel with standard deviation α.

Two distinct characteristics of NLM can be confirmed from the formulation in Eq. (5):

(1) The weights are based on the dissimilarities between patches.
(2) The restored gray value of each pixel i is obtained from a non-local search window S_i. For the sake of computation, a 21×21 window centered in pixel i is commonly recommended in [4,5,15].

2.3. Introduction to MNF

Due to totally different image degrading mechanisms brought out by Gaussian noise and impulse noise, up to date, just few works focus on the removal of the mixture of the two noises, though such a mixture is common in the real world. To make NLM capable to remove such mixed noise, Li et al. designed a mixed noise filter (MNF) [10] by incorporating rank-ordered absolute difference statistic (ROAD) into NLM, where the ROAD value is used to quantify how different in intensity the specific pixel is from its most similar neighbors (refer to [9] for more details). As a result, the output of MNF can be represented as

$$\hat{X}(i) = \frac{1}{C(i)} \sum_{j \in S_i} w_I(j) \exp\left(\frac{||Y_i - Y_j||^2_{2,M}}{2\sigma_M^2} \right) Y(j) \tag{8}$$