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Unified detection of skewed rotation, reflection and translation symmetries from affine invariant contour features

Zhaozhong Wang [a,*], Lianrui Fu [a], Y.F. Li [b]

[a] Image Processing Center, Beihang University, Beijing 100191, China
[b] Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, China

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ABSTRACT

Symmetry detection is significant for object detection and recognition since symmetries are salient cues for distinguishing geometrical structures from cluttered backgrounds. This paper proposes a unified framework to detect rotation, reflection and translation symmetries simultaneously on unsegmented real-world images. The detection is performed based on affine invariant contour features, and is applicable under skewed imaging with distortions. Contours on a natural image are first matched to each other to find affine invariant matching pairs, which are then classified into three groups using a *sign change* criterion corresponding to the three types of symmetries. The three groups are used to vote for the corresponding symmetries: the voting for rotation utilizes a scheme of *short line segments*; the voting for reflection is based on a parameterization of axis equation, and the voting for translation uses a cascade-like approach. Experimental results of real-world images are provided with quantitative evaluations, validating that the proposed framework achieves desired performance.

1. Introduction

Detections of rotation, reflection and translation symmetries increasingly find their use in pattern recognition, 2D image processing, and 3D object reconstruction. Their applications include but not limited to: photo editing [1] (using reflection), object recognition [2] (using reflection and rotation), image segmentation [3] (using reflection), scene perception [4] (using translation), digital inspection [3] (using reflection and rotation), scene reconstruction [5] (using translation), and 3D object reconstruction [6] (using reflection).

This paper aims to detect the three common symmetries simultaneously for a potentially wide variety of applications. The contribution of this paper is that it detects the three types of symmetries of natural images under skewed imaging in a unified framework, which is novel in the aspects that: (1) it uses contour features on unsegmented real-world images to detect all the symmetries; (2) it is based upon affine invariant contour matching; (3) it proposes a *sign change* criterion to group the matching pairs; and (4) it adopts novel voting schemes to detect different symmetries.

Though detection methods based on ideal contours on synthetical images are well studied, those for cluttered contours on natural images are few to be researched. Using affine invariant contour matching on natural images we can unify the detection of rotation, reflection and translation in the same way, since they are special cases of the affine transform. By the grouping scheme we classify the single affine transform to three types for different voting process. To simplify the work we only vote for the major aspects of symmetries: for rotation symmetries we detect their rotation centers; for reflection symmetries we detect their symmetry axes; and for translation symmetries we detect their near-regular lattices. The pipeline of the algorithm thus includes four main stages: contour detection, contour matching, contour grouping and contour voting for symmetries.

In the following sections, we first analyze related work on symmetry detection in Section 2, then propose the theoretical formulation and algorithm framework in Sections 3 and 4, respectively. The algorithm implementation and evaluation methods are given in Section 5, and experimental results are provided in Section 6. The conclusion are given in the last section.

2. Related work

Recent surveys for symmetry detection [7,8] indicate that after several decades of effort, symmetry detection in real-world images still remains a challenging problem. We shall analyze existing work for detecting rotation, reflection, translation symmetries and multiple of them, with remarks on contour-based methods.

* Corresponding author. Tel.: +86 10 8233 8578; fax: +86 10 8231 6502.
E-mail address: zwang@buaa.edu.cn (Z. Wang).

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Rotation symmetry detection has been well researched [9–18]. Some previous work only applied to non-skewed rotation symmetry detection [9–11], but skewed rotation symmetry is much more common in real-world cases. Cornelius and Loy [13] adopted local invariant features, such as SIFT, for skewed rotation symmetry detection. Yip [14] utilized the Hough transform for the detection of rotation under parallel projection. Carlsson [15] provided a general mathematical characterization of rotation symmetry in perspective space. Lee and Liu [12] proposed a frieze-expansion method that transforms rotation symmetry detection into a simple translation symmetry detection problem. There is relatively little literature on contour-based rotation symmetry detection: Yip [16] proposed a genetic Fourier descriptor to detect rotation from partially occurred or opened contours. Liu et al. [17] proposed a dihedral and frieze group detection algorithm for paper-cut patterns using edge features. Venkatesh and Cheung [18] proposed an algorithm for rotationally symmetric shape completion in the presence of severe occlusions. Most of the contour-based algorithms only detect rotation symmetry from synthetic or well segmented images. Our proposed algorithm, however, tries to detect rotation symmetry directly from natural images with noisy backgrounds.

There are a number of approaches that have been proposed for reflection symmetry detection. Researches for non-skewed reflection symmetry detection [19–21] mainly use methods related to the Fourier transform. Skewed reflection symmetry has been studied not only for synthetic images [22,23] but also for real-world images [24,25]. The features for reflection symmetry detection varies from key points such as SIFT [24–26] to contours [23,27,28]; we mainly focus on review of the latter. Early contour-based reflection detection can be found in [27], where a symmetry-enhancing edge detector was presented to find symmetric objects. Marola [23] applied affine invariant feature vector that captures both local and semi-local geometric features around each point of the object boundary. Masuda et al. [28] utilized image gradients and edge pixel matching to find lines of symmetry. Bruckstein and Shaked [29] proposed an approach to detect skewed reflection based on the theory of invariant signatures for planar objects. Tek and Kimia [30] presented an approach based on analytic geometry computations and numerical solutions of PDE's to detect the symmetry of skeleton from contour segments. The contour-based algorithms above mainly handle well segmented objects with closed contours, or images with clean backgrounds. Methods for natural images still need to be improved.

Existing work on translation symmetry detection has achieved impressive results for either regular [31] or near-regular [32–35] textures. Lin et al. [31] detected the periodicity of regular textures based on autocorrelation functions. Liu et al. [32] proposed methods to model and construct near-regular patterns of the textures in images. Zhao and Quan [33] proposed the MRF optimization method which can identify and separate multiple symmetries from images with mild skews. Hays et al. [34] proposed a higher-order feature matching algorithm to detect the lattices of near-regular textures in real images. Park et al. [35] formulated the 2D lattice detection as a spatial, multi-target tracking problem and solved it within an MRF framework using a mean-shift belief propagation method. They [4] also proposed a mid-level visual feature detector where the visual elements are grouped based on the 2D translation subgroup of a wallpaper pattern. To the best of our knowledge, few existing algorithms have attempted to characterize translation symmetry from contour features on real images. Our approach in this paper makes an attempt to detect translation symmetry using a unified local affine invariant contour matching framework.

There are also methods proposed to detect more than one types of symmetries, e.g., both rotation and reflection, but almost none of them could cover all the three types of symmetries under a single framework. Tuytelaars et al. [36] presented a geometric framework for the detection of rotation and reflection symmetries through the Hough transform. Loy and Eklundh [37] proposed a pairwise local feature matching algorithm using SIFT key points at corresponding locations to detect reflection and rotation, which could be extended for translation. Chertok and Keller [38] presented a spectral approach for detecting and analyzing rotation and reflection symmetries in 2D images and 3D synthetic objects. Kondra et al. [39] proposed multi-scale symmetry kernel operators for reflection and rotation. Additionally there exist some contour-based algorithms [40,41] introducing reflection and rotation detection under a single framework. Lei and Wong [40] utilized a Hough-based detector to find symmetry axis from edge contours. Their algorithm is global and is sensitive to clutters in the background. Guo et al. [41] presented a set of quantitative symmetric measures of irregular shapes based on a series of geometric transformation operations. Their algorithm is originally developed for galaxy classification and is limited to well segmented objects with closed contours. Our proposed method aims to establish the framework to detect all the three types of symmetries, using both closed and open contours on unsegmented images.

3. Symmetry formulations

In this section we discuss from the original formulation of symmetry to the proposed formulation by affine transformations to detect symmetries under distortions.

3.1. Original formulation of symmetries

An object is *symmetric* with respect to a given operation if the operation leaves the object unchanged, and the operation is called a *symmetry operation* [42]. The set of symmetry operations taken together often forms a *group*, so symmetry is typically defined by the group theory. A *symmetry group* of an object is the group of all *isometries* under which the object is invariant. An *isometry* of a n-dimensional space $\mathbb{R}^n$ is a function $f_{\text{iso}}: \mathbb{R}^n \rightarrow \mathbb{R}^n$ that preserves distance, that is,

$$\| f_{\text{iso}}(\mathbf{x}) - f_{\text{iso}}(\mathbf{x}') \| = \| \mathbf{x} - \mathbf{x}' \|, \quad \mathbf{x}, \mathbf{x}' \in \mathbb{R}^n,$$

where $\| \cdot \|$ is a norm on the space. In the 2D Euclidean plane $\mathbb{R}^2$, there are totally five types of isometries: identity, rotation, reflection, translation, and glide-reflection.

Except the trivial type of identity, the rotation, reflection, and translation are common symmetries, which are invariant to their corresponding transformations. A glide-reflection symmetry means a reflection with respect to a line combined with a translation along the line. It is as important as the pervious three, and generally appears in periodic-contents to classify wallpaper images [43]. Though the glide-reflection symmetry is not detected in this work, we shall consider it as a future work to improve the proposed framework.

3.2. Symmetries formulated by affine transformations

To detect the rotation, reflection and translation symmetries in a unified framework, it is a key to detect the corresponding symmetry operations (isometries). To this purpose we shall detect first a single affine transformation. There are two reasons for this treatment. First, the three operations of rotation, reflection and translation are all special cases of the affine transformation $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$,

$$f(\mathbf{x}) = A\mathbf{x} + \mathbf{t}, \quad \mathbf{x} \in \mathbb{R}^2, \tag{1}$$