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# Some hybrid weighted averaging operators and their application to decision making ☆

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ABSTRACT

Two new hybrid weighted averaging operators for aggregating crisp and fuzzy information are proposed, some of which desirable properties are studied. These operators helps us to overcome the drawback in the existed reference. With respect to the proposed operators, three special types of preferred centroid of triangular fuzzy number are defined. On the base of these preferred centroid, we develop two algorithms to deal with decision making problems. Two numerical examples are provided to illustrate the practicality and validity of the proposed methods.



## 1. Introduction

The ordered weighted averaging (OWA) operator [1] is useful for aggregating a finite collection of arguments, and has been receiving more and more attention over the last decades. It provides a parameterized family of aggregation operators that include the minimum, maximum, and average as special cases. Based on the geometric mean, Herrera and Viedma [2] developed the ordered weighted geometric (OWG) operator to aggregate the arguments in a similar way as the OWA operator. Chen et al. [3] indicated that the ordered weighted harmonic (OWH) operator is to aggregate central tendency date. All the above operators consider only the ordered position of the argument, the given importance of the argument is not taken into account. In order to reflects both the given importance and ordered position of the argument, Xu and Da [4] introduced the hybrid weighted averaging (HWA) operator. Since its appearance, many hybrid aggregation operators have been proposed, such as the linguistic hybrid geometric averaging (LHGA) operator [5], the uncertain linguistic hybrid aggregation (ULHA) operator [6], the induced generalized hybrid averaging (IGHA) operator [7], the generalized intuitionistic fuzzy hybrid aggregation (GIFHA) operator [8], and so on. However, the HWA operator does not satisfy boundary and idempotent which are desirable for aggregating a finite collection of arguments. Therefore, we proposed a new hybrid aggregation operator here to overcome some drawbacks of the HWA operator.

Recently, Yager [9] developed a continuous ordered weighted arithmetic averaging (C-OWA) operator to aggregate all the values in a closed interval, and the weights of C-OWA operator are determined by a basic unit-interval monotonic (BUM) function. The C-OWA operator is appropriate for aggregating decision information which are given in the forms of valued interval. Moreover, the continuous ordered weighted geometric averaging (C-OWG) operator [10] and the continuous ordered weighted harmonic averaging (C-OWH) operator [11] are proposed based on geometric mean and harmonic mean, respectively. Zhou and Chen [12] developed the continuous generalized OWA (C-GOWA) operator to adapt to uncertain and complex situations in decision making. Wu et al. [13] introduced the induced continuous ordered weighted geometric averaging (ICOWG) operator, which is able to contain the reciprocity and consistency properties of the collective preference relation. Especially, they present the relative consensus degree induced COWG operator and the reliability induced COWG operator. Moreover, they study some desirable properties of the ICOWG operator in decision making problems [14]. Nevertheless, the above-mentioned operators only solve the ordered position aggregation over a closed interval $[a,b]$, the aggregation of given importance of $[a,b]$ is not taken into account. In this paper, some new aggregation operators are developed to solve this problem, and two methods for decision making are developed to illustrate the application of the proposed operators.

The paper is organized as follows. Section 2 introduces some preliminary knowledge. In Section 3, two new hybrid aggregation operators called HWAA operator and HWQA operator are

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proposed. Section 4 extends the HWQA operator to the continuous HWQA operator. In Section 5, two methods for decision making are developed. Section 6 provides two numerical examples to demonstrate the practicality and validity of the proposed methods. The paper is concluded in Section 7.

## 2. Preliminaries

For the convenience of analysis, some basic concepts on aggregation operators are introduced to facilitate future discussions.

**Definition 2.1.** [15]. A weighted arithmetical averaging (WAA) operator of dimension n is a mapping WAA: $R^n \to R$, according to the following formula:

$$WAA(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} \lambda_i a_i \tag{2.1}$$

where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ is the weighting vector of the real numbers $a_1, a_2, \ldots, a_n$, such that $\sum_{i=1}^{n} \lambda_i = 1$ and $\lambda_i \in [0,1]$, $R$ is the set of real numbers.

The ordered weighted averaging (OWA) operator was introduced by Yager [1], which provide a parameterized family of aggregation operators, and can be defined as follows:

**Definition 2.2.** An OWA operator of dimension n is a mapping OWA: $R^n \to R$, defined by an associated weighting vector $W = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$, according to the following formula:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i a_{\sigma(i)} \tag{2.2}$$

where $\sigma$: $\{1,2,\ldots,n\} \to \{1,2,\ldots,n\}$ is a permutation such that $a_{\sigma(i)} \geqslant a_{\sigma(i+1)}$, $\forall i = 1, 2, \ldots, n-1$. Namely, $a_{\sigma(i)}$ is the $i$th largest element of the collection of real numbers $a_1, a_2, \ldots, a_n$, $R$ is the set of real numbers.

On the other hand, (2.2) can be equivalently written as

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_{\sigma^{-1}(i)} a_i \tag{2.3}$$

where $\sigma^{-1}$: $\{1,2,\ldots,n\} \to \{1,2,\ldots,n\}$ is the inverse permutation of $\sigma$. $a_i$ is the $\sigma^{-1}(i)$th largest element of the collection of real numbers $a_1, a_2, \ldots, a_n$. Let $\varepsilon = \sigma^{-1}$, then (2.3) can also be written as

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_{\varepsilon(i)} a_i \tag{2.4}$$

It clear that $a_i$ is the $\varepsilon(i)$th largest element of the collection of real numbers $a_1, a_2, \ldots, a_n$.

Based on the OWA operators and the geometric mean, Herrera and Viedma [2] investigated the ordered weighted geometric averaging (OWG) operator, which can be defined in the following:

**Definition 2.3.** An OWG operator of dimension n is a mapping OWG: $R^{+n} \to R^+$, defined by an associated weighting vector $W = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$, shown as follows:

$$OWG(a_1, a_2, \ldots, a_n) = \prod_{i=1}^{n} b_i^{w_i} \tag{2.5}$$

where $b_i$ is the $i$th largest element of the collection of real numbers $a_1, a_2, \ldots, a_n$, $R^+$ is the set of positive real numbers.

In order to aggregate central tendency data, the ordered weighted harmonic averaging (OWH) operator [3] is proposed as follows.

**Definition 2.4.** An OWH operator of dimension n is a mapping OWH: $R^n \to R$, defined by an associated weighting vector $W = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$, according to the following formula:

$$OWH(a_1, a_2, \ldots, a_n) = \frac{1}{\sum_{i=1}^{n} \frac{w_i}{a_{\sigma(i)}}} \tag{2.6}$$

where $\sigma$: $\{1,2,\ldots,n\} \to \{1,2,\ldots,n\}$ being a permutation such that $a_{\sigma(i)} \geqslant a_{\sigma(i+1)}$, $\forall i = 1, 2, \ldots, n-1$.

Fodor [16] replaced the above-mentioned means with quasi-arithmetical averaging, and proposed the QOWA operator as follows.

**Definition 2.5.** A QOWA operator of dimension n is a mapping QOWA: $R^n \to R$, defined by an associated weighting vector $W = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$, and a continuous strictly monotonic function $h$, according to the following formula:

$$QOWA(a_1, a_2, \ldots, a_n) = h^{-1}\left(\sum_{i=1}^{n} w_i h(b_i)\right) \tag{2.7}$$

where $b_i$ is the $i$th largest element of the collection of the real numbers $a_1, a_2, \ldots, a_n$. $h^{-1}$ is the inverse function of $h$.

By combining the advantages of the WAA operator and the OWA operator, Xu [4] proposed the hybrid weighted averaging (HWA) operator, shown as follows:

**Definition 2.6.** A HWA operator of dimension n is a mapping HWA: $R^n \to R$, defined by an associated weighting vector $W = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$, according to the following formula:

$$HWA(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i \tag{2.8}$$

where $b_i$ is the $i$th largest of the weighted arguments $n\lambda_i a_i$ $(i = 1,2,\ldots,n)$ and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ is the weighting vector of the $a_i$ $(i = 1,2,\ldots,n)$, with $\sum_{i=1}^{n} \lambda_i = 1$ and $\lambda_i \in [0,1]$, and $n$ is the balancing coefficient.

The HWA operator generalizes both the OWA and WAA operators and reflects both the given importance and the ordered position of the arguments. However, the HWA operator does not satisfies some desirable properties, such as boundary, idempotent, etc. For example, assume $(a_1, a_2, a_3, a_4) = (1,1,1,1)$ and $W = \lambda = (1,0,0,0)^T$, then $HWA(1,1,1,1) = 4 \times 1 = 4 > 1 = \max_i\{a_i\}$.

Recently, in order to aggregate all the values in a closed interval $[a,b]$, Yager [9] developed a continuous ordered weighted arithmetic averaging (C-OWA) operator based on OWA operator and BUM function.

**Definition 2.7.** A continuous ordered weighted arithmetic averaging operator is a mapping f: $\Omega \to R$ which is defined as following:

$$f_Q([a,b]) = \int_0^1 \frac{dQ(y)}{dy}[b - (b-a)y]dy \tag{2.9}$$

where $Q$ is a basic unit-interval monotonic (BUM) function $Q$: $[0,1] \to [0,1]$ and is monotonic with the properties: (1) $Q(0) = 0$; (2) $Q(1) = 1$; and (3) $Q(x) \geqslant Q(y)$ if $x > y$. $\Omega$ is the set of closed intervals.

**Remark 2.1.** When the BUM function $Q$ is a step function, the Definition 2.7 is still valid [9]. For example,