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A Lattice-Computing ensemble for reasoning based on formal fusion of disparate data types, and an industrial dispensing application

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ABSTRACT

By "fusion" this work means integration of disparate types of data including (intervals of) real numbers as well as possibility/probability distributions defined over the totally-ordered lattice $(\mathsf{R},\leqslant)$ of real numbers. Such data may stem from different sources including (multiple/multimodal) electronic sensors and/or human judgement. The aforementioned types of data are presented here as different interpretations of a single data representation, namely Intervals' Number (IN). It is shown that the set F of INs is a partially-ordered lattice $(\mathsf{F},\preceq)$ originating, hierarchically, from $(\mathsf{R},\leqslant)$. Two sound, parametric *inclusion measure* functions $\sigma:\mathsf{F}^N\times\mathsf{F}^N\to[0,1]$ result in the Cartesian product lattice $(\mathsf{F}^N,\preceq)$ towards decision-making based on reasoning. In conclusion, the space $(\mathsf{F}^N,\preceq)$ emerges as a formal framework for the development of hybrid intelligent fusion systems/schemes. A fuzzy lattice reasoning (FLR) ensemble scheme, namely *FLR pairwise ensemble*, or *FLRpe* for short, is introduced here for sound decision-making based on descriptive knowledge (rules). Advantages include the sensible employment of a sparse rule base, employment of granular input data (to cope with imprecision/uncertainty/vagueness), and employment of all-order data statistics. The advantages as well as the performance of our proposed techniques are demonstrated, comparatively, by computer simulation experiments regarding an industrial dispensing application.

1. Introduction

In the domain of *Soft Computing* or, equivalently, *Computational Intelligence*, the term "hybrid (system/algorithm)" frequently denotes an integration of different techniques/technologies including artificial neural networks, fuzzy systems, evolutionary/swarm computing, etc. towards improving an index of performance in real-world applications [1,15]; the term "intelligence" is pertinent to decision-making, e.g. in pattern classification/recognition [82]; moreover, the term "(intelligent) fusion" may signify an aggregate intelligence towards improving decision-making [48]. In the aforementioned sense, a "hybrid intelligent fusion system" may be a Multiple Classifier System (MCS) [46,49] also known in the literature as *Classifier Ensemble* [16,59,65] or *Committee* [21,80] or *Voting Consensus* [5,51]. Note that a number of MCS architectures/strategies including applications have been reported [22,29,30,47,50,52,55,56,70,71,74,81,85,86]. The MCS techniques are, typically, of statistical nature [34] in the Euclidean space R^N. Nevertheless, a "hybrid intelligent fusion system" may be defined otherwise, as explained in the following.

The term "fusion" may, alternatively, denote an integration of data stemming from multiple, even heterogeneous, sources including (multimodal) electronic devices as well as human judgement [6,9,13,17,20,27,53,57,64,66,68]. In the latter context, there is a keen interest in formal frameworks for unified decision-making based on disparate types of data that may accommodate uncertainty [9,18,79]. One such a framework has been proposed lately [36], in an information engineering context, based on mathematical *lattice theory* as follows.

Different authors have recognized that several types of data of practical interest, including information *granules* [62,84], are partially(lattice)-ordered [38,72]. Hence, lattice theory emerged as a formal framework for the fusion of disparate data types [36]. In such context, *fuzzy lattice reasoning* (FLR) was originally proposed [37,42,44] as a specific rule-based scheme for classification in a complete lattice $(\mathsf{L},\preceq)$ data domain including, as a special case, the lattice of hyperboxes in the Euclidean space R^N. In this work, FLR is defined, more widely, as any employment of an *inclusion measure* function $\sigma:\mathsf{L}\times\mathsf{L}\to[0,1]$ for decision-making. Therefore, in the context of this work, the term "intelligent" is pertinent to "FLR (reasoning)".

Instead of a general mathematical lattice this work considers a specific one originating hierarchically from the totally-ordered lattice $(\mathsf{R},\leqslant)$ of real numbers. Note that the latter (lattice) has stemmed, historically, from the conventional measurement process of successive comparisons [36,42]. Our interest in lattice $(\mathsf{R},\leqslant)$ was motivated by the existence of vast quantities of real number measurements stored worldwide. Therefore, we sought

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convenient data/information representations based on R. Hence, the complete lattice $(\mathsf{F},\preceq)$ of Intervals' Numbers (INs) emerged, as detailed below, where a IN is a unified data representation including real numbers, intervals, and probability/possibility distributions [60]. In conclusion, the Cartesian product lattice $(\mathsf{F}^N,\preceq)$ is introduced here as a formal framework for developing hybrid intelligent fusion systems/schemes, where an element of lattice $(\mathsf{F}^N,\preceq)$ is interpreted as either a rule (of a FLR scheme) or as an input to a FLR scheme.

In previous work, a FLR scheme for classification has been implemented on the σ-FLNMAP neural network architecture [36,43,45]. Note that the latter (neural network) architecture was introduced as an enhancement of the fuzzy-ARTMAP, or FAM for short, neural classifier [11]. More specifically, the σ-FLNMAP has extended the applicability domain of FAM from the lattice of hyperboxes in R^N to any complete lattice data domain. Moreover, note that even in the Euclidean space R^N, that is FAM's sole "applicability domain", classifier σ-FLNMAP retains, comparatively, significant advantages including the capacity to introduce tunable nonlinearities as well as the capacity to deal with both non-overlapping hyperboxes and granular (hyperbox) input data [36,43].

Due to the fact that both classifiers σ-FLNMAP and FAM are *unstable*, in the sense that their testing accuracy depends on the order of presenting the training data [19,43], both σ-FLNMAP and FAM make good candidates for *Voting* classification schemes [10,36,69]. Indeed, empirical studies have clearly demonstrated an improved testing accuracy as well as a more stable testing accuracy for both FAM [3,12,61] and σ-FLNMAP [36,45] in R^N. Later work has extended the applicability of σ-FLNMAP from the lattice of hyperboxes to the lattice $(\mathsf{F},\preceq)$ of INs based on FLR [42]. In all, FLR is a *Lattice-Computing* scheme as explained next.

Lattice-Computing (LC) is a term introduced by Graña [23] to denote any computation in a mathematical lattice. Graña and colleagues have demonstrated a number of LC techniques in image processing applications [24–26]; in particular, they have employed mathematical morphology techniques in the totally-ordered lattice of real numbers. It turns out that FLR is also a LC scheme, in particular for "reasoning" as explained below.

This paper is based on previously published work on FLR including the following novelties. First, it presents a space of INs as a formal information fusion framework including a large number of references as well as pertinent discussions – a novel mathematical proof is also introduced here. Second, it includes mathematical notation improvements. Third, it introduces an enhanced definition of FLR. Fourth, it demonstrates "in principle" an accommodation of granular inputs. Fifth, it introduces a novel decision-making scheme, that is a descriptive (rule-based) FLR ensemble of experts. Sixth, it shows a number of illustrative, new examples including figures. Seventh, it demonstrates preliminary (computer simulation) results regarding an industrial application.

The layout of this work is as follows. Section 2 presents a formal framework for fusion/integration of disparate data types. Section 3 describes our proposed FLR ensemble scheme. Section 4 outlines an industrial application. Section 5 demonstrates, comparatively, preliminary experimental results. Section 6 concludes by summarizing our contribution. The Appendix presents novel mathematical notation as well as a novel mathematical proof.

2. A formal information fusion framework

This section introduces constructively, in four steps, a formal information fusion framework, namely the Cartesian product lattice $(\mathsf{F}^N,\preceq)$ of Intervals' Numbers (INs). Different interpretations of INs are also presented. Note that the four-level hierarchy of lattices presented here is a novelty of this work. For the interested reader, useful notions and tools regarding lattice theory are summarized in the Appendix.

2.1. The complete lattice $(\overline{\mathsf{R}},\leqslant)$

The set R of real numbers is a totally-ordered, non-complete lattice denoted by $(\mathsf{R},\leqslant)$. It turns out that $(\mathsf{R},\leqslant)$ can be extended to a complete lattice by including both symbols "$-\infty$" and "$+\infty$". In conclusion, the complete lattice $(\overline{\mathsf{R}},\leqslant)$ emerges, where $\overline{\mathsf{R}} = \mathsf{R}\cup\{-\infty,+\infty\}$. Note that in previous work we, erroneously, assumed that lattice $(\mathsf{R},\leqslant)$ is complete [38,60]. Even though the aforementioned error is not critical, this work considers, instead, the complete lattice $(\overline{\mathsf{R}},\leqslant)$ [1]. We remark that complete lattices are important not only in defining an *inclusion measure* function, as shown in the Appendix, but they are also important in *mathematical morphology* [58,67].

On the one hand, any strictly increasing function $v:\overline{\mathsf{R}}\to\mathsf{R}$ is a positive valuation in the complete lattice $(\overline{\mathsf{R}},\leqslant)$. Motivated by the two constraints presented in the Appendix (subsection B), here we consider positive valuation functions $v:\overline{\mathsf{R}}\to\mathsf{R}^{\geqslant 0}$ such that both $v(-\infty)\doteq lim_{x\to-\infty}v(x)=0$ and $v(+\infty)\doteq lim_{x\to+\infty}v(x)=A<+\infty$. On the other hand, any *bijective* (i.e. one-to-one) strictly decreasing function $\theta:\overline{\mathsf{R}}\to\overline{\mathsf{R}}$ is a dual isomorphic function in lattice $(\overline{\mathsf{R}},\leqslant)$. We will refer to functions $\theta(\cdot)$ and $v(\cdot)$ as *dual isomorphic* and *positive valuation*, respectively. Useful extensions to the corresponding lattice of intervals are presented next.

2.2. The complete lattice $(\Delta,\preceq)$ induced from $(\overline{\mathsf{R}},\leqslant)$

A *generalized interval* is defined in lattice $(\overline{\mathsf{R}},\leqslant)$ as follows.

Definition 1. *Generalized interval* is an element of the product lattice $(\overline{\mathsf{R}},\leqslant^{\partial})\times(\overline{\mathsf{R}},\leqslant)$.

Recall that $\leqslant^{\partial}$ in Definition 1 denotes the *dual* (i.e. converse) of order relation $\leqslant$ in lattice $(\overline{\mathsf{R}},\leqslant)$, i.e. $\leqslant^{\partial}\equiv\geqslant$. The product lattice $(\overline{\mathsf{R}},\leqslant^{\partial})\times(\overline{\mathsf{R}},\leqslant)\equiv(\overline{\mathsf{R}}\times\overline{\mathsf{R}},\geqslant\times\leqslant)$ will be denoted, simply, by $(\Delta,\preceq)$.

A generalized interval will be denoted by $[x,y]$, where $x,y\in\overline{\mathsf{R}}$. It follows that the *meet* $(\curlywedge)$ and *join* $(\curlyvee)$ in lattice $(\Delta,\preceq)$ are given, respectively, by $[a,b]\curlywedge[c,d]=[a\vee c,b\wedge d]$ and $[a,b]\curlyvee[c,d]=[a\wedge c,b\vee d]$.

The set of *positive* (*negative*) generalized intervals $[a,b]$, characterized by $a\leqslant b$ $(a>b)$, is denoted by Δ_+ (Δ_-). It turns out that $(\Delta_+,\preceq)$ is a poset, namely *poset of positive generalized intervals*. Note that poset $(\Delta_+,\preceq)$ is *isomorphic* to the poset $(\tau(\overline{\mathsf{R}}),\preceq)$ of conventional intervals (sets) in $\overline{\mathsf{R}}$, i.e. $(\tau(\overline{\mathsf{R}}),\preceq)\cong(\Delta_+,\preceq)$. We augmented poset $(\tau(\overline{\mathsf{R}}),\preceq)$ by a *least* (empty) interval, denoted by $O=[+\infty,-\infty]$ – we remark that a *greatest* interval $I=[-\infty,+\infty]$ already exists in $\tau(\overline{\mathsf{R}})$. Hence, the complete lattice $(\tau_O(\overline{\mathsf{R}})=\tau(\overline{\mathsf{R}})\cup\{O\},\preceq)\cong(\Delta_+\cup\{O\},\preceq)$ emerged. In the sequel, we will employ isomorphic lattices $(\Delta_+\cup\{O\},\preceq)$ and $(\tau_O(\overline{\mathsf{R}}),\preceq)$, interchangeably. We point out that a trivial interval $[x,x]$ is an *atom* in the complete lattice $(\tau_O(\overline{\mathsf{R}}),\preceq)$; where an atom $[x,x]$, by definition, satisfies both $[+\infty,-\infty]=O\prec[x,x]$ and there is no interval $[a,b]\in(\tau_O(\overline{\mathsf{R}}),\preceq)$ such that $O\prec[a,b]\prec[x,x]$.

Consider both a positive valuation function $v:\overline{\mathsf{R}}\to[0,A]$, where $0<A<+\infty$, and a dual isomorphic function $\theta:\overline{\mathsf{R}}\to\overline{\mathsf{R}}$. Then, Proposition 6.2 implies that function $v_\Delta:\Delta\to\mathsf{R}$ given by $v_\Delta([a,b])=v(\theta(a))+v(b)$ is a positive valuation in lattice $(\Delta,\preceq)$. There follow both $v_\Delta(O=[+\infty,-\infty])=0$ and $v_\Delta(I=[-\infty,+\infty])<+\infty$. Therefore, based on Theorem 6.1 (in the Appendix), the following two inclusion measures emerge in lattice $(\Delta,\preceq)$

[1] Personal communication with Peter Sussner in the context of the Hybrid Artificial Intelligence Systems (HAIS '2010) International Conference, 23–25 June 2010, San Sebastian, Spain. It is understood that the authors here assume full responsibility for possible errors.