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On the impact of anisotropic diffusion on edge detection

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ABSTRACT

Content-aware, edge-preserving smoothing techniques have gained visibility in recent years. However, they have had a rather limited impact on the edge detection literature compared to content-unaware (linear) techniques, often based on Gaussian filters. In this work, we focus on Anisotropic Diffusion, covering its initial definition by Perona and Malik and subsequent extensions. A visual case study is used to illustrate their features. We perform a quantitative evaluation of the performance of the Canny method for edge detection when substituting linear Gaussian smoothing filters by Anisotropic Diffusion.

1. Introduction

Edge detection is often based on the analysis of intensity differences within pixel neighbourhoods. Intuitively, this leads to the computation of the partial derivatives or the Laplacian of a signal [1,2]. However, due to the discrete nature of the data, the classical concept of differentiation cannot be applied. The ill-posedness of the gradient computation is partially solved by regularizing (smoothing) the image. The simplest and most common way to do so is content-unaware smoothing (CUS), which is usually performed using two-dimensional Gaussian filters [3,4]. CUS techniques remove noise and image imperfections at the cost of blurring the edges [5]. For this reason, there is a need for smoothing techniques that regularize the pixel intensities within the image objects, while preserving (or even sharpening) their boundaries. As enunciated by Monteil and Beghdadi [6], the goal is to combine *a low pass filtering in homogeneous regions and a sharpening effect in transition regions*.

Content-aware smoothing (CAS) techniques adjust their behaviour based upon local features. One of the most relevant CAS techniques is *Anisotropic Diffusion* (AD), initially proposed by Perona and Malik [5], who formulate the smoothing problem in terms of heat diffusion. They were aiming at a process in which the image properties (in this case, intensity) would spread inside the objects but not across their boundaries. In this way, AD allows heat (intensity) diffusion inside the objects, inhibiting the heat transfer across the edges. Following [5], different approaches to AD have been explored, incorporating notions from statistics [7], fuzzy logic [8], and photometry [9], among others. Moreover, AD has proven valid not only for image regularization, but also as inpainting method in the reconstruction of missing (deleted) information [10–12].

Despite the variety of CAS techniques (either based on AD or not), most of the works on edge detection still use linear filtering [13,14] or do not even regularize the image [15,16]. One of the reasons is the lack of quantitative comparisons confronting CUS and CAS techniques. Indeed, most of the works proposing CAS for edge detection contain sample images where the improvements can be observed, but do not quantify the results. The few works using objective measures [17,18] are related to noise removal. Note that the lack of quantitative comparisons is not exclusive to CAS proposals, but rather endemic to the edge detection field [19], to such an extent that Papari and Petkov state that quantitative comparison is *absent from most of the works in edge detection* [20].

In this work we take AD as a paramount example of CAS, and we list three objectives:

(a) to briefly analyze the technical specifications of the relevant AD methods for edge detection,
(b) to illustrate the effect of such AD methods on natural images, and
(c) to quantify the improvement that can be achieved by a typical edge detector if replacing Gaussian linear filtering by any of the aforementioned AD methods.

We approach these tasks from a practical point of view. First, we review the most relevant AD methods in the literature, pointing out their connection with the original proposal by Perona and

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Malik, as well as listing their parameters and settings. Second, we apply different AD methods to a natural image, in order to observe and compare the changes the image undergoes. Third, we perform a quantitative comparison of the results produced by the Canny method for edge detection [3] when combining it with Gaussian Linear Filtering (GLF) and different AD methods. In this work we consider the scalar approach to AD [5,7], as well as approaches based on vectorial gradients [21–23] and speckle models [18,24].

The remainder of this work is organized as follows. Section 2 contains a review of the literature. In Section 3 we analyze how different AD methods use local information to drive the diffusion process. Section 4 includes a visual example of the transformation of an image when different AD methods are applied. To conclude, the experimental results and conclusions are presented in Sections 5 and 6, respectively.

2. Anisotropic Diffusion

Anisotropic Diffusion is an example of *scale-space* image processing [5,22,25,26]. It stems from the application of the heat diffusion equation to digital images. Considering an image I, the heat flux ϕ within the image is given by Fick's equation

$$\phi = -D \cdot \nabla I, \tag{1}$$

where the relationship between the gradient ∇I and the actual flux j is expressed by a positive definite, symmetric matrix D referred to as *diffusion tensor*. Since the diffusion process does not alter the overall energy in the image, its local variation is driven by $\delta_t I = -\mathrm{div}\phi$ [22], where div stands for the divergence operator. Hence, we have that

$$\delta_t I = \mathrm{div}(D \cdot \nabla I) \tag{2}$$

expresses the energy (heat) variation at every position in the image. The abstraction in Eq. (2) has been embodied in different ways, leading to the creation of a wide family of AD methods. In this work we focus on five such methods, which we consider representative:

(i) *Perona–Malik Anisotropic Diffusion*: The first reference to the application of AD to digital images is due to Perona and Malik [5]. The authors consider the diffusion process on an image I to be modelled by

$$\delta_t I = \mathrm{div}(g(|\nabla I|^2) \cdot \nabla I), \tag{3}$$

where the function g is an edge-stopping, decreasing function weighing the conductivity of the image depending upon the Euclidean magnitude of the gradient. However, they propose a simpler, discrete scheme based on the transfer of energy between each pixel and its four direct neighbours. Despite its simplicity, this scheme has some interesting properties such as the preservation of the energy or the fact that it does not create new (local) maxima or minima. We will refer to this method as Perona–Malik AD (PMAD).
Almost at the same time, Saint-Marc et al. [27] introduced the notion of adaptive smoothing, which turns out to be an alternative implementation of PMAD (see [28] for a historical perspective). Catté et al. [29] proposed to compute the gradients on a regularized version of the image I, such as $I_\sigma = G_\sigma * I$, where G_σ represents a Gaussian filter with standard deviation σ and $*$ is the convolution operator. In this way, the authors claim to solve the *inconsistencies* of PMAD, more specifically the fact that small changes in the initial image *can produce divergent solutions* [29]. Subsequently, many authors have revisited PMAD, mainly focusing on the instability of the process and the staircasing effect (creation of non-existing step edges) [30–32].

(ii) *Diffusion Tensor-based Anisotropic Diffusion*: Cottet and Germain [23] argue against the use of the name AD in [5], since PMAD makes use of scalar conductivity values instead of diffusion tensors. From Eq. (2) we obtain isotropic and non-linear isotropic diffusion using

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{or } D = \begin{pmatrix} g(|\nabla I|) & 0 \\ 0 & g(|\nabla I|) \end{pmatrix}, \tag{4}$$

respectively [33]. In order to obtain an anisotropic behaviour, we must use the direction of ∇I, not only its magnitude. Diffusion Tensor-based AD (DTAD) is based on Eq. (2), where the eigenvectors of D are $v_1 \| \nabla I_\sigma$ and $v_2 \perp \nabla I_\sigma$. In order not to smoothen across the edges, the eigenvalues are set to $\lambda_1 = g(|\nabla I_\sigma|^2)$ and $\lambda_2 = 1$ [22].

(iii) *Structure Tensor-based Anisotropic Diffusion*: A *structure tensor* (J_ρ) is a symmetric matrix associated with a gradient ∇I_σ constructed as

$$J_\rho(\nabla I_\sigma) = G_\rho * (\nabla I_\sigma \otimes \nabla I_\sigma). \tag{5}$$

The structure tensor is useful to characterize the underlying structure and features of the image, not only the local intensity variations. It differs from the diffusion tensor in the fact that it captures the orientation of the intensity change, not its direction. The integration parameter ρ has to be set according to the size of the underlying image structure [22]. Note that σ and ρ play similar roles, although they have different purposes. The parameter σ aims to remove (or reduce the impact of) noise and imperfections in the image, and hence is related to the contamination of the image, not to its content. Alternatively, ρ takes values according to the size of the structures and object silhouettes one wants to preserve. When using structure tensors, we replace the matrix D in Eq. (2) by a matrix with the same eigenvectors as J_ρ and eigenvalues

$$\lambda_1(\mu_1) = g(\mu_1) \quad \text{and} \quad \lambda_2 = 1 \tag{6}$$

where μ_1 is the greatest eigenvalue of J_ρ. The AD method based on structure tensors is referred to as STAD.

(iv) *Coherence Enhancing Anisotropic Diffusion*: The purpose of Coherence Enhancing AD (CEAD) [34] is intrinsically different from that of the other AD methods. In this approach the smoothing is not prevented across the edges, but empowered along them instead. That is, there is heat transfer in edge regions, but it takes place along the edges. In this approach the matrix D in (2) is replaced by a matrix with the same eigenvectors μ_1 and μ_2 as the structure tensor J_ρ and eigenvalues

$$\lambda_1 = \alpha \quad \text{and} \quad \lambda_2 = \begin{cases} \alpha, & \text{if } \mu_1 = \mu_2 \\ \alpha + (1-\alpha) \cdot g(\mu_1 - \mu_2), & \text{otherwise} \end{cases}, \tag{7}$$

where $\alpha \in]0, 1[$ is a regularization parameter that ensures a certain amount of diffusion to occur in situations with isotropic intensity change, and takes a positive value close to 0 [21].

(v) *Speckle Reducing Anisotropic Diffusion*: Some authors have presented AD methods whose local behaviour is not determined by gradients. A relevant example is the methods based on noise estimation, such as Speckle Reducing Anisotropic Diffusion (SRAD), introduced by Yu and Acton [18]. Speckle noise is a very common type of contamination in digital images. It is manifested as random impulse noise introducing normally distributed variations in the pixels of an image. SRAD intends to regularize the image in the presence of speckle, while not carrying out regularization in homogeneous regions or edge regions. In SRAD