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Image representation using separable two-dimensional continuous and discrete orthogonal moments

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ABSTRACT

This paper addresses bivariate orthogonal polynomials, which are a tensor product of two different orthogonal polynomials in one variable. These bivariate orthogonal polynomials are used to define several new types of continuous and discrete orthogonal moments. Some elementary properties of the proposed continuous Chebyshev–Gegenbauer moments (CGM), Gegenbauer–Legendre moments (GLM), and Chebyshev–Legendre moments (CLM), as well as the discrete Tchebichef–Krawtchouk moments (TKM), Tchebichef–Hahn moments (THM), Krawtchouk–Hahn moments (KHM) are presented. We also detail the application of the corresponding moments describing the noise-free and noisy images. Specifically, the local information of an image can be flexibly emphasized by adjusting parameters in bivariate orthogonal polynomials. The global extraction capability is also demonstrated by reconstructing an image using these bivariate polynomials as the kernels for a reversible image transform. Comparisons with the known moments are performed, and the results show that the proposed moments are useful in the field of image analysis. Furthermore, the study investigates invariant pattern recognition using the proposed three moment invariants that are independent of rotation, scale and translation, and an example is given of using the proposed moment invariants as pattern features for a texture classification application.

1. Introduction

Image moments have been shown to be useful in image analysis [1,2], image watermarking [3], and invariant pattern recognition [4–5]. These moments include continuous orthogonal Legendre moments (LLM) [6] and discrete orthogonal moments (e.g., Tchebichef moments (TTM) [7], Krawtchouk moments (KKM) [8], Hahn moments (HHM) [9–11]). All of these known two-dimensional orthogonal moments have separable basis functions that can be expressed as two separate terms by producing two *same* classical orthogonal polynomials each of one variable. Such classical orthogonal polynomials of a single variable have been long studied [12–14].

Multivariate orthogonal polynomials are important research topics for pure mathematics and mathematical physics. Especially in recent years, bivariate orthogonal polynomials have attracted considerable research interest as solutions of second-order partial differential equations [15,16]. The general method of generating bivariate continuous orthogonal polynomials from univariate continuous orthogonal polynomials can trace its origins back to [17]. In [17], Koornwinder introduces seven different classes of orthogonal polynomials in two variables and gives some examples of two variable analogs of the Jacobi polynomials. In addition to Jacobi polynomials, bivariate continuous orthogonal polynomials as extensions of other univariate continuous orthogonal polynomials have also been reported in [18,19]. Dunkl and Xu [20] give an excellent review of bivariate discrete orthogonal polynomials as the tensor product of two families of classical discrete orthogonal polynomials of one variable in their highly regarded work.

The applications of bivariate or multivariate orthogonal polynomials are evident in various areas of applied mathematics, but only a few papers are involved in their application for image analysis and pattern recognition. The current work has been inspired by the tensor product of two orthogonal polynomials in one variable as proposed by [16,17,21–24]. By producing an orthogonal polynomials method, this paper presents several continuous and discrete orthogonal moments with bivariate orthogonal polynomials as basis functions. Unlike the traditional basis functions that are tensor products of two *same* continuous (or discrete) orthogonal polynomials in one variable, the present study shows that the tensor products of two *different* or *same* orthogonal polynomials in one variable, such as product Gegenbauer polynomials, product Chebyshev polynomials, product Chebyshev and Gegenbauer polynomials, product Gegenbauer and Legendre polynomials, product Chebyshev and Legendre

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polynomials, product Tchebichef and Krawtchouk polynomials, product Tchebichef and Hahn polynomials, and product Krawtchouk and Hahn polynomials, can also be used to generate basis functions for two-dimensional orthogonal moments, which are denoted GGM, CCM, CGM, GLM, CLM, TKM, THM, and KHM, respectively. To the best of our knowledge, no such basis functions have been used to define the two-dimensional moments. The objective of this paper is to introduce such bivariate orthogonal polynomials into the field of image analysis and attempt to demonstrate their usefulness in this field. In this study, we emphasize that such nomenclature is derived from leading literature [6] where Teague proposed orthogonal polynomials as the basis or kernel functions for moment calculations; different polynomials lead to various types of moments.

The accuracy of proposed moments as global descriptors is assessed by reconstructing the whole image and subsequently comparing to reconstructions using the known Legendre, Tchebichef, Krawtchouk, and Hahn moments. In addition, the study investigates the local image representation capabilities of the proposed moments for images by adjusting the parameters of bivariate polynomials. Because the parameters of orthogonal polynomials are set as special values, the emphasized region of these polynomials is completely different. Thus, by producing a polynomials method, a few proposed two-dimensional moment functions (e.g., TKM, THM, and KHM) gain the ability to extract the features of any select region of the image by choosing appropriate parameters and give additional flexibility in describing the image. In contrast to non-negative matrix factorization (NMF) [25,26], our descriptor can capture the local information of an image by adjusting the polynomial parameters. NMF attempts to find two non-negative matrices with product that can well approximate the original data and achieves part-based representation.

As an important task in pattern recognition, texture classification has been the subject of extensive research during the last several years [27,28]. The key step in any texture classification process is the choice of the set of features used to characterize the texture, and moment invariants are highly popular sets that are widely used in various classification tasks. In this work, we use the proposed moments that are invariant under 2-D transforms, such as rotation, scaling, and translation, as features for texture classification. Because the proposed moments are expressed as linear combinations of the geometric moments, we can derive three moment invariants using the method applied to Krawtchouk moments by Yap et al. [8]. The texture classification accuracy of the proposed moment invariants is also compared to the existing moment invariants. The experimental results indicate that the proposed descriptors have better discriminative power than methods based on geometric or complex moments.

The main contributions of this work include the following four aspects. (1) The tense product of two *different* univariate polynomials can construct basis function for defining moments. (2) The paper derives three moment invariants that are independent of the transformations involving rotation, scale and translation. (3) By producing two *different* polynomials, some proposed moments have the local feature extraction capability. (4) The proposed method can be easily extended to obtain other bivariate orthogonal polynomials and their corresponding moments.

The rest of this paper is structured as follows. In Section 2, we discuss the known results of the classical continuous and discrete orthogonal polynomials of a variable; these previous studies serve as basic background for the rest of this work. Sections 3 and 4 are devoted to introducing continuous and discrete orthogonal polynomials in two variables. Section 5 presents the two-dimensional orthogonal moments. Section 6 focuses on the deriving moment invariants from the geometric moments. Section 7 demonstrates a few examples. Finally, concluding remarks can be found in Section 8.

2. Classical orthogonal polynomials of a variable

For completeness, we have included the classical orthogonal polynomials of a variable to serve as a foundation for the rest of the work. These polynomials were previously obtained in [12–14].

2.1. Classical continuous orthogonal polynomials

2.1.1. Jacobi polynomials

The family of Jacobi polynomials $J_n^{(\alpha,\beta)}(x)$ satisfies the following orthogonality relation:

$$\int_{-1}^{1}(1-x)^{\alpha}(1+x)^{\beta}J_m^{(\alpha,\beta)}(x)J_n^{(\alpha,\beta)}(x)dx = \frac{2^{\alpha+\beta+1}}{2n+\alpha+\beta+1}\frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{n!\Gamma(n+\alpha+\beta+1)}\delta_{mn}, \quad m,n\geq 0, \tag{1}$$

with the known conditions $\alpha>-1$, and $\beta>-1$, and the recurrence relation:

$$xJ_n^{(\alpha,\beta)}(x) = \frac{2(n+1)(n+\alpha+\beta+1)}{(2n+\alpha+\beta+1)(2n+\alpha+\beta+2)}J_{n+1}^{(\alpha,\beta)}(x) + \frac{\beta^2-\alpha^2}{(2n+\alpha+\beta+2)(2n+\alpha+\beta)}J_n^{(\alpha,\beta)}(x) + \frac{2(n+\alpha)(n+\beta)}{(2n+\alpha+\beta)(2n+\alpha+\beta+1)}J_{n-1}^{(\alpha,\beta)}(x). \tag{2}$$

2.1.2. Gegenbauer polynomials

The Gegenbauer polynomials [12] $G_n^{(\lambda)}(x)$ are Jacobi polynomials with $\alpha=\beta=\lambda-1/2$, which satisfies the following orthogonality relation:

$$\int_{-1}^{1}(1-x^2)^{\lambda-(1/2)}G_m^{(\lambda)}(x)G_n^{(\lambda)}(x)dx = \frac{\pi\Gamma(n+2\lambda)2^{1-2\lambda}}{(\Gamma(\lambda))^2(n+\lambda)n!}\delta_{mn}, \tag{3}$$

with the known conditions, $\lambda>-(1/2)$, $\lambda\neq 0$.

Gegenbauer orthogonal polynomials satisfy a recurrence relation of the form:

$$2(n+\lambda)xG_n^{(\lambda)}(x) = (n+1)G_{n+1}^{(\lambda)}(x)+(n+2\lambda-1)G_{n-1}^{(\lambda)}(x), \quad G_0(x)=1 \text{ and } G_1(x)=2\lambda x. \tag{4}$$

The normalized Gegenbauer polynomials $\tilde{G}_n^{(\lambda)}(x)$ are defined as

$$\tilde{G}_n^{(\lambda)}(x) = \sqrt{\frac{(1-x^2)^{\lambda-1/2}(\Gamma(\lambda))^2(n+\lambda)n!}{\pi\Gamma(n+2\lambda)2^{1-2\lambda}}}G_n^{(\lambda)}(x). \tag{5}$$

2.1.3. Chebyshev polynomials

The Chebyshev polynomials of the first kind [12] $C_n(x)$ can be obtained from the Jacobi polynomials by taking $\alpha=\beta=1/2$, which satisfies the following orthogonality relation:

$$\int_{-1}^{1}(1-x^2)^{-(1/2)}C_m(x)C_n(x)dx = \begin{cases} \frac{\pi}{2}\delta_{mn}, & n\neq 0 \\ \pi\delta_{mn}, & n=0 \end{cases}. \tag{6}$$

The Chebyshev polynomials of the first kind satisfy a recurrence relation of the form:

$$2xC_n(x) = C_{n+1}(x)+C_{n-1}(x), \quad C_0(x)=1 \text{ and } C_1(x)=x. \tag{7}$$

The normalized Chebyshev polynomials $\tilde{C}_n(x)$ are defined as

$$\tilde{C}_n(x) = \sqrt{\frac{2}{\pi}}(1-x^2)^{-(1/4)}C_n(x), \quad n\neq 0. \tag{8}$$