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An effective dual method for multiplicative noise removal

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ABSTRACT

The problem of multiplicative noise removal has been widely studied recently, but most models focus on the unconstrained problems. These models require knowing the prior level of noise beforehand, however, the information is not obtained in some case and the regularization parameters are not easy to be adjusted. Thus, in the paper, we mainly study an optimization problem with total variation constraint, and propose two new denoising algorithms which compute the projection on the set of images whose total variation is bounded by a constant. In the first algorithm, we firstly give the dual formula of our model, then compute the dual problem using alternating direction method of multipliers. Experimental results show that our method is simple and efficient to filter out the multiplicative noise when the prior of noise is unknown.

1. Introduction

Image denoising is one of the fundamental problems in the image processing and computer version fields. A real recorded image may be distorted by many expected or unexpected random factors, of which random noise is an unavoidable one. Most of the literatures deal with the additive noise model: given an original image f, it is assumed that it has been corrupted by the additive noise η. The goal is then to recover f from the data $f_0 = f + \eta$. Many approaches have been proposed for this problem [1–3].

In this paper, we are concerned with the issue of multiplicative noise removal. Specifically, we are interested in removing the Gamma distributed multiplicative noise from a contaminated image. The multiplicative model is

$$f_0 = f\eta, \tag{1}$$

where $f_0 > 0$ is the observed image, $f > 0$ is the original image, and η is the noise which follows a Gamma Law with mean one and its probability density function is given by

$$g(\eta) = \frac{M^M}{\Gamma(M)}\eta^{M-1}\exp(-M\eta)1_{\{\eta>=0\}}. \tag{2}$$

where M is the number of looks (in general, an integer coefficient), and $\Gamma(\cdot)$ is a Gamma function. Multiplicative noise is one of the more complex image noise models. It is signal independent, non-Gaussian, and spatially dependent. Hence, multiplicative noise removal is a very challenging problem compared with additive Gaussian noise.

Multiplicative noise removal methods have been discussed in many reports. Popular methods include the Lee method [4], multiscale shrinkage and Bayesian MAP estimator methods for the log-data [5–7], various variational methods [8–18], and the augmented Lagrangian approach [19]. In particular, among the variational methods, the total variation (TV) based methods have become very popular in the recent few years. The first TV based multiplicative noise removal model was presented by Rudin et al. [16], which used a constrained optimization approach with two Lagrange multipliers. Following the maximum a posteriori (MAP) estimator for multiplicative Gamma noise, Aubert and Aujol [9] introduced a non-convex model

$$\min_{f\in BV(\Omega)}\left\{\|f\|_{TV} + \alpha_1\int_\Omega\left(\log f + \frac{f_0}{f}\right)dx\right\}, \tag{3}$$

where $\|f\|_{TV} = \int_\Omega|\nabla f|dx$ is the regularization term, $\int_\Omega\left(\log f + \frac{f_0}{f}\right)dx$ is the data fidelity term, and α_1 is the regularization parameter.

Although the above two models have obtained some relatively good results, but their corresponding algorithms have a slower rate of convergence because of the nonconvexity of their fidelity terms. In order to overcome this drawback, recently, Steidl and Teuber [8] introduced a new variational restoration model consisting of the I-divergence as data fidelity term and the TV as regularizer

$$f = \arg\min_{f\in BV(\Omega),\, f>0}\left\{\lambda\|f\|_{TV} + \int_\Omega f - f_0 + f_0\log\frac{f_0}{f}dx\right\}, \tag{4}$$

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where the data fidelity term $\int_\Omega f - f_0 + f_0 \log\frac{f_0}{f}\,dx$ is strictly convex, λ is a regularization parameter. The authors computed the minimizer of Eq. (4) by applying the Douglas–Rachford splitting techniques, resp. alternating split Bregman method, combined with an efficient algorithm to solve the involved nonlinear systems of equations (see the details in [8]). However, the regularization parameter λ is related to the level of noise. If the prior of noise is not clear for us, it is not easy to adjust the parameter λ for achieving the better restoration results.

In this paper, we mainly focus on removing the multiplicative noise from the image f_0 by solving the following optimization problem with a bounded TV constraint

$$\min_{\|f\|_{TV}\leqslant\tau, f>0} \int_\Omega f - f_0 + f_0 \log\frac{f_0}{f}\,dx, \quad \text{where} \quad \tau < \|f_0\|_{TV}. \tag{5}$$

Compared with Eq. (4), the model (5) has an important advantage: parameter τ has a clear meaning (it is proportional to the TV of f) and is much easier to set than the parameter λ which is determined by the noise level σ, so the model (5) might be preferable over Eq. (4) when more is known about the TV estimation τ than the noise level σ. The problem with this kind of constrain for additive noise was recently studied in [20]. To the best of our knowledge, the multiplicative noise removal problem with this constrain has never been studied before.

The rest of the paper is arranged as follows. In Section 2, we introduce some basic tools that will be used in the paper. In Section 3, the proposed model (5) is discussed in detail and one dual algorithm is given. In Section 4, we give another algorithm based on split and duality for the proposed model. In Section 5, we give some numerical results to show the effectiveness of our method. Finally, conclusions are given in Section 6.

2. Preliminaries

Let Ω be a two-dimensional bounded open domain of R^2 with Lipschitz boundary, then an image can be interpreted as a real function defined on Ω. In this section, we first review some basic definitions and notations that will be used in this paper.

Definition 2.1. Let Du be the distributional derivative of u, We define $BV(\Omega)$, the space of functions of bounded variation, as

$$BV(\Omega) = \left\{ u \in L^1(\Omega); \quad \int_\Omega |Du| < \infty \right\}.$$

Definition 2.2. Let C be a nonempty convex set. The indicator function χ_C of C is

$$\chi_C(p) = \begin{cases} 0, & \text{if } p \in C, \\ +\infty, & \text{otherwise}. \end{cases}$$

Definition 2.3. Let $\varphi : H \to R$ be a proper, lower-semicontinuous and convex function, where H is a Hilbert space. Then, for every $q \in h$, the function $h \mapsto \phi(h) + \|q-h\|_2^2/2$, for $h \in H$, achieves its infimum at a unique point denoted by

$$\text{prox}_{\gamma\phi}(q) = \arg\min_h \left\{ \frac{1}{2}\|h-q\|_2^2 + \gamma\phi(h) \right\},$$

where γ is a constant value.

In the discrete case, an image f of $N = n \times n$ pixels can be seen as a vector in R^N. We use $\|\cdot\|_2^2$ to denote the norm induced by the inner product $\langle\cdot,\cdot\rangle$ in R^N, and define $\|f\|_\infty = \max_{i,j}|f(i,j)|$. Let $X = R^N \times R^N$ be the space of vector fields with the inner product

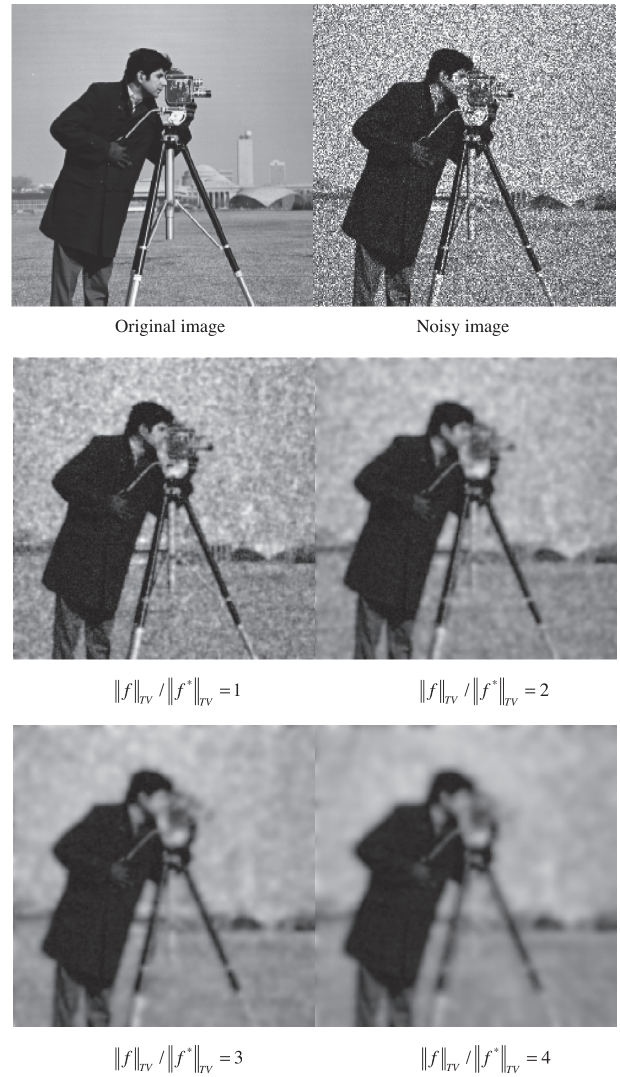

Fig. 1. The denoising results computed with our Algorithm 1.

$$\langle u, v\rangle_X = \sum_{i=1}^n \sum_{j=1}^n u_1(i,j)v_1(i,j) + u_2(i,j)v_2(i,j), \quad \text{for} \quad \forall \quad u, v \in X,$$

where $u = (u_1, u_2)$, $v = (v_1, v_2)$. The l^1 and l^∞ norms of a vector field $u = (u_1, u_2) \in X$ are respectively

$$\|u\|_1 = \sum_{i=1}^n \sum_{j=1}^n |u(i,j)| \quad \text{and} \quad \|u\|_\infty = \max_{i,j}|u(i,j)|,$$

where $|u(i,j)| = \sqrt{u_1(i,j)^2 + u_2(i,j)^2}$. The discrete gradient of $f \in R^N$ is defined as $\nabla f(i,j) = (\nabla_x f(i,j), \nabla_y f(i,j)) \in X$, with

$$\nabla_x f(i,j) = \begin{cases} f(i+1,j) - f(i,j), & \text{if } 1 \leqslant i < n, \\ 0, & \text{otherwise}, \end{cases}$$

$$\nabla_y f(i,j) = \begin{cases} f(i,j+1) - f(i,j), & \text{if } 1 \leqslant j < n, \\ 0, & \text{otherwise}. \end{cases}$$

The discrete total variation of f is $\|f\|_{TV} = \|\nabla f\|_1$, where the l^1 norm of a vector field in X has been defined above. The adjoint of the gradient is $\nabla^* = -\text{div}$, and the discrete divergence of a vector field $u = (u1, u2) \in X$ is $\text{div}(u) = \partial_x u_1 + \partial_y u_2$, with