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Short Communication

A rectilinear Gaussian model for estimating straight-line parameters

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For characterizing straight lines in defocused images, a rectilinear Gaussian model (RGM) is proposed. Based on this model, a novel method for estimating the parameters of straight lines is presented. This method, called gray-scale least square (GLS) method, directly deals with gray-scale image data without requiring any preprocessing and hence no additional noise is introduced. Furthermore, the method is able to simultaneously estimate four parameters of straight lines by performing the algorithm only once, while two parameters can be typically estimated by traditional method. Besides this, all parameters are given in closed-form solution. In order to illustrate the effectiveness of RGM and the GLS method, the experiments are performed on a set of artificial images and natural images. The experimental results show that the GLS method outperforms the traditional method from the point of view of sensitivity to noise and accuracy of parameter estimation.

1. Introduction

Lines in an image can be defined as global abstractions from local intensity changes. They are highly distinguishable features in images and convey the most relevant information of imaging conditions. For this reason, lines have been widely employed in a large variety of scientific fields and application fields. For example, they serve as matching primitives in stereo matching [1,2], camera calibration [3] and object recognition [4,5]. They also tend to act as focus points in automatic focus [6]. Such lines are either straight or curve, however, our discussion will be confined to straight lines, and strictly speaking, to line segments. This is partly due to the fact that line segments are more robust in providing greater positional accuracy than curved lines, which is crucial especially for precision measurement and high-accuracy positioning. At the same time, curved lines can be approximated as aggregates of piecewise-linear segments at a suitable scale [7–9].

Regardless of the task, if the straight-line parameters are effectively estimated, a variety of other subsequent processing steps are greatly facilitated. For this reason, many of methods have been proposed to extract straight lines and estimate parameters. To the best of our knowledge, almost all methods can estimate only two parameters (i.e., orientation and distance). However, a line segment should be considered not as a group of collinear pixels in the image [10] but as a separate geometric structure that can be fully defined by five parameters, which are: length, width, orientation, distance, and maximum intensity on center. These parameters play a key role in such tasks as image matching and their relative importance varies between tasks. For instance, in stereo matching and engineering drawings, length, width and maximum intensity are employed as more important factors among these parameters [2,11]; width and maximum intensity play a more important role in automatic focus[6], while orientation and distance are more important parameters for the tasks of registration and rectification [12].

Generally, the estimation of straight-line parameters is composed roughly of two basic steps: (1) detect edge points by edge detectors such as the Canny algorithm [13]; (2) estimate the parameters of straight lines by the Hough transform (HT) [14,15] or least square (LS) [16]. The actual step taken depends on the specific algorithm. For example, the HT method in the second step acts directly on the whole image without requiring partitioning, while the LS method needs to determine the line-support region with a metric such as proximity and similarity of orientation [7] before parameter estimation. Unfortunately, in many cases, a single real edge results in several strong edge responses at different (often parallel) locations [8]. Moreover, edge detectors frequently misplace or entirely miss edges due to low signal-to-noise ratio or low intensity contrast. At the same time, the information of intensity in an input image, which is very important for parameter estimation, is removed by edge detectors in the first step. The problems in the discussion on edge operators pose difficulties for parameter estimation with high accuracy. To resolve this dilemma, some of the pioneering work in such areas was performed by Lo

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and Tsai [17]. The method to directly estimate straight-line parameters from gray-scale image data was designed, however, it is inherently very expensive computationally due to 4D parameter counting space. Alternatively, Zheng et al. [18] proposed a more efficient method based on the Radon transform, whereas the six parameters on which the resolution depends are required to be specified artificially.

Furthermore, the performance of the approaches based on either HT or LS is progressively degraded with an increase in the amount of defocus in the imaging process. At the same time, the depth of field is limited and hence the image captured by camera is frequently blurred [19], especially for the cameras with shallow focus [20]. In fact, precise focus is possible at only one distance, that is, the image of a point in 3D space is sharp only when the point intersects a particular hyperboloid [21,22], and the imaged objects become increasingly blurred with increase of the distance between the imaged point and the hyperboloid of exact focus. In this sense, strictly speaking, defocus of a scene is inevitable, especially for microscopic imaging. It is worth noting that defocus carries important information for parameter estimation [21,23].

Based on the aforementioned considerations, we developed an approach to straight-line parameters estimation by exploiting defocus information, but the approach proposed here is not limited to defocused lines, which is proven by testing. Due to the fact that in many cases, the canonical representation will be the starting point for obtaining the optimal solution, a reasonable line model is necessary to be designed exclusively for accurate parameter estimation. For this purpose, we proposed RGM, by means of which a desired method of estimation for straight-line parameters (i.e., the GLS method) is presented.

The remainder of this paper is organized as follows. In Section 2, we discuss related work. In Section 3, the details of RGM are described. The GLS method, which is an application of RGM to straight-line parameter estimation, is described in detail in Section 4. Experimental results are given in Section 5. Some concluding remarks are given in Section 6.

2. Related work

In this section, we discuss the previous works related to straight lines, mainly focusing on parameter estimation, which is one of the basic tasks in a large variety of scientific fields [24–27]. A considerable number of approaches to straight-line parameter estimation have been devised towards this end. These approaches fall into two broad categories: One is the category based on HT or, more generally, the Radon transform; the other is based on the LS method. The HT method exploits a "one to many" transform, in which one edge point is transformed to many points in the parameter space. As a result, inherent shortages in memory space and computational costs exclude such methods from many real world applications. For this reason, the HT method is not covered here and an extensive review can be found in [14,27,28]. Instead, we will focus on the LS method in the following section.

2.1. The LS method

The LS method is one of the widely used techniques and the basic method to fit lines to points in the plane [16], which was first used by Gauss to calculate definitive orbits of solar system bodies in 1795 [29]. For the purpose of discrimination, the method is known as ordinary least squares (OLS), which can be represented as follows:

Given an $m \times n$ real matrix A of rank $k < \min(m,n)$, and an m-dimensional vector B, suppose the linear regression model is $B = A\beta + e$, the LS problem is how to find an n-dimensional vector β so that the Euclidean length of $A\beta - B$ reaches minimum value.

In the OLS problem, there is an underlying assumption that all the errors are confined to the observation vector B. Unfortunately and frequently, both A and B are simultaneously noisy, therefore, the assumption mentioned above is violated. This leads the solution β_{LS} to suffer from great bias and covariance. To overcome this problem, Pearson [30] proposed the total least squares (TLS) method, which is considered as a natural generalization of the LS method. The TLS method defines the residue as the normal (or shortest) distance of the point to the line instead of the difference of the y coordinates [16]. However, the vector β will be a strongly consistent estimate only if the measurement errors for each observation are independently and identically distributed with zero mean. When this condition is also violated, parameter estimation must employ the structured total least squares (STLS) algorithm [31,32] or the constrained total least squares (CTLS) approach [33]. In fact, the STLS approach is equal to the CTLS approach, which is proven by Lemmerling et al. [34].

In spite of its mathematical beauty and computational simplicity, one single outlier (points that are far from the line [35]) generated by random noise can have an arbitrarily large effect on the LS estimators, no matter how big the sample size is [36]. This lack of stability of the LS method is a serious problem in applications [37]. Hence, robust alternatives to the method of least squares are sorely needed. To alleviate the problem, Huber [38] introduced an M estimator based on a maximum likelihood estimate. However, the breakdown point (the maximum fraction of outliers which a given sample may contain without spoiling the estimate completely [36,39]) of the M estimator is zero because of the possibility of leverage points. To remedy this situation, generalized M estimators [40,41] were introduced. Unfortunately, they have a breakdown point tending to 0 when the dimension of data matrix A increases [39]. The S-estimator [42] is the first estimator that attained the maximum breakdown point. However, it cannot simultaneously achieve a high breakdown point and high efficiency [37,43]. The MM-estimator [39] and τ-estimator [44] obtain robust and efficient estimations. Unfortunately, this comes at the cost of an increase in bias [37].

Many researchers have drawn on robust and efficient estimations another way. Agostinelli and Markatou [45] presented the weighted least squares (WLS) estimator, but their weighting scheme is complicated. Gervini and Yohai [37] proposed the robust and efficient weighted least squares (REWLS) estimator based on the WLS estimator, which is asymptotically efficient if errors are normally distributed. However, the asymptotic distribution of REWLS typically depends upon the initial estimator. To address this issue, Čížek [46] proposed the two-step least weighted squares (2S-LWS) estimator, which has an asymptotic distribution independent of the initial estimate and preserves its robust properties.

The least median of squares (LMS) estimator, which was first proposed by Hampel [47], replaces the sum in the OLS estimator by a median to attain maximum breakdown point. Contrary to OLS, LMS has no closed-form solution, and therefore it must be solved by iterative algorithms. Furthermore, its objective function requires sorting of the squared residuals. For this reason, the initial LMS estimator has a very low efficiency. To improve the efficiency of the LMS estimator, Rousseeuw [36] suggested using a high breakdown-point estimate followed by a one-step M-estimate or a one-step re-weighted least squares. However, its exact breakdown point is not known and therefore it is not clear whether the breakdown-point of the initial estimate is changed [39]. In fact, the LMS estimate is equivalent to finding the strip defined by two parallel lines of minimum vertical separation that encloses at least half of the points. The best algorithm for finding the LMS strip is the topological plane-sweep algorithm due to Edelsbrunner and