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First-order modeling and stability analysis of illusory contours

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Abstract

In visual cognition, illusions help elucidate certain intriguing latent perceptual functions of the human vision system, and their proper mathematical modeling and computational simulation are therefore deeply beneficial to both biological and computer vision. Inspired by existent prior works, the current paper proposes a first-order energy-based model for analyzing and simulating illusory contours. The lower complexity of the proposed model facilitates rigorous mathematical analysis on the detailed geometric structures of illusory contours. After being asymptotically approximated by classical active contours, the proposed model is then robustly computed using the celebrated level-set method of Osher and Sethian [S. Osher, J.A. Sethian, Fronts propagating with curvature-dependent speed: algorithms based on Hamilton–Jacobi formulations, J. Comput. Phys., 79 (12) (1988) 12–49] with a natural supervising scheme. Potential cognitive implications of the mathematical results are addressed, and generic computational examples are demonstrated and discussed.

Keywords: Illusion; Illusory contours; Energy model; Local stability; Contour decomposition; Real and imaginary; Kinks; Spans; Turns; Level-set method; Supervision

1. Introduction

In system theory [22], input–output analysis has been a major tool for partial or complete identification of black-box systems. In cognitive vision science, the study of various visual illusions follows exactly the same spirit. Cognitive scientists have designed numerous intriguing inputs of image signals, so that the distorted or transformed outputs (as reported by an average human observer) can help reveal some crucial latent properties of the human vision system (e.g. the remarkable works [1,15,17,12]). *Illusory contours* are such a well known class of visual illusions, and the current paper develops a mathematical model to characterize, analyze, and simulate generic illusory contours. Our work has been closely inspired by many existent modeling works, especially by Sarti et al. [27], and Zhu and Chan [37,38].

We set out by emphasizing first that by no means the current mathematical model puro be absolutely faithful in the context of neuron science or cortical vision. Rather, the present work still remains *phenomenological*, as [27,37,38] do. In fact, it is precisely our ultimate goal to be able to develop a neurally rational model that incorporates all the important discoveries by neuron scientists (e.g. [18,34,33,36,25]). The current work and all the aforementioned inspiring predecessors may thus pave the valuable stepping stones towards this goal.

Fig. 1 shows two examples of illusory contours known as *Kanizsa triangle and square* [12,27,37]. The illusory or *imaginary* white triangle and square pop out almost instantly to a normal observer. The human vision system is capable of interpolating non-existent edges and making

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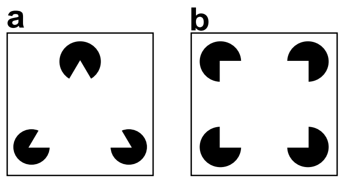

Fig. 1. Two classical examples of illusory contours: Kanizsa's triangle and square.

meaningful visual organization of both the *real* and *imaginary* edge segments. From the viewpoints of modeling and computation, such illusory perception has also been called *segmentation with missing boundaries* [27], and is closely related to some earlier works (e.g. [20,21]).

Qualitatively, the perception of illusory contours can be deciphered by the Gestalt framework [11,12]. In a typical example of illusory contours, the illusory target objects share the same gray level as their background and become invisible to ordinary detectors. The illusory targets however do leave their footprints, mainly through the cues of occlusion, e.g. the visible corners in Fig. 1. An average human vision system can organize these cues and develop a so-called 2.1-D sketch of the scene [21], i.e., to separate and complete objects in ordered (according to the relative depth to a viewer) multiple layers. The multi-layer sketch of Kanizsa's triangle is displayed in Fig. 2, for example. We also refer the reader to the recent work of Shen [32] on an abstract study of the occlusion phenomenon and 2.1-D models.

Quantitatively, however, it is a non-trivial task to properly model and compute illusory contours. There are two notable recent works that have primarily influenced the present work in developing plausible mathematical models and robust computational schemes.

In [27], Sarti et al. first proposed a variational-PDE model for computing illusory contours based on a *reference point* within an image. Given a reference point which models the focus of visual attention, a surface is then constructed and flattened except along the existing *real* edges. The level sets of the surface function are able to connect the imaginary contours. Define the edge indicator function by [27]

$$g(x) = g(x_1, x_2) = \frac{1}{1 + (|\nabla G_\sigma(x) * u(x)|/\beta)^2},$$

$$G_\sigma(\xi) = \frac{\exp(-(\xi/\sigma)^2)}{\sigma\sqrt{\pi}}.$$

Then the model is to minimize the g-weighted area of the surface S: $x = (x_1, x_2) \to (x_1, x_2, \Phi)$:

$$\min_\Phi \int_\Omega g(x)\sqrt{1 + \Phi_{x_1}^2 + \Phi_{x_2}^2}\, \mathrm{d}x.$$

Our model shares a similar computational formulation, but is more self-contained in terms of modeling and analysis since it is directly built on curves rather than on reference points and surfaces. (We must point out that, in visual cognition, attention focuses do play important roles in a number of cognitive tasks [12].)

Later in [37], Zhu and Chan proposed a more complex level-set-based variational model. Let $u : \Omega \to \mathbb{R}$ be an image defined on the domain Ω, with $u = 1$ on the objects and $u = -1$ otherwise. The key component of Zhu and Chan's model is the *signed distance* function from solving the eikonal equation:

$$\frac{\partial d}{\partial t} = \mathrm{sign}(u)(1 - |\nabla d|),$$

with the initial data $d(x, t = 0) = u(x) = u(x_1, x_2)$. Their variational model is then to minimize the following energy for the level-set function ϕ which codes the desired illusory contours:

$$E(\phi) = \int_\Omega ((1 + \mu C(d)\kappa^+(d))|d|\delta(\phi)|\nabla\phi| + \lambda H(d)H(\phi))\, \mathrm{d}x + \int_\Omega \left(a + b\left|\nabla \cdot \left[\frac{\nabla\phi}{|\nabla\phi|}\right]\right|^2\right)|\nabla\phi|\delta(\phi)\, \mathrm{d}x, \qquad (1)$$

where δ denotes Dirac's delta, H the Heaviside function, and C a C^1 differentiable cut-off function. The $|d|\delta(\phi)|\nabla\phi|$ term measures the distance from the zero level-set to the existent boundaries and $C(d)\kappa^+(d)|d|\delta(\phi)|\nabla\phi|$ measures the distance to concave corners. The second term of the first integral measures the overlap between the existent objects and the interior of ϕ. The last integral represents Euler's elastica energy which was first proposed by Mumford [19], and later applied to geometric image inpainting by Chan et al. [3], and Esedoglu and Shen [9]. Zhu and Chan [37] showed outstanding performance of this model for several general examples. Our proposed model is much less complex than this high-order geometric model, and is also independent of the signed distance function. The essence of our proposed model is to be able to capture the most salient features of illusory perception in a simple and analyzable manner, at the unsurprising cost of losing certain high-order details. (Zhu and Chan [38] later also

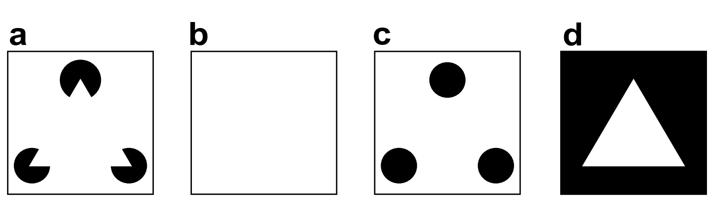

Fig. 2. Multi-layer decomposition of Kanizsa's triangle: (a) image of Kanizsa's triangle, (b) far background, (c) three disks in the middle range, and (d) the illusory triangle closest to a viewer.