Available online at www.sciencedirect.com

ScienceDirect

Pattern Recognition 40 (2007) 335–338

PATTERN RECOGNITION
THE JOURNAL OF THE PATTERN RECOGNITION SOCIETY

www.elsevier.com/locate/patcog

Rapid and brief communication

Subspace evolution analysis for face representation and recognition

Huahua Wang, Yue Zhou*, Xinliang Ge, Jie Yang

Institute of Image Processing and Pattern Recognition, Shanghai Jiao Tong University, Shanghai 200240, PR China

Received 22 January 2006; accepted 7 June 2006

Abstract

This paper develops a novel framework that is capable of dealing with small sample size problem posed to subspace analysis methods for face representation and recognition. In the proposed framework, three aspects are presented. The first is the proposal of an iterative sampling technique. The second is adopting divide–conquer–merge strategy to incorporate the iterative sampling technique and subspace analysis method. The third is that the essence of 2D PCA is further explored. Experiments show that the proposed algorithm outperforms the traditional algorithms.

Keywords: PCA; 2D PCA; LDA; Iterative sampling technique; Divide-conquer-merge

1. Introduction

Subspace analysis methods, such as PCA and LDA, have become the most successful approaches in face recognition [1,2]. In conventional PCA-based or LDA-based face recognition method, the image is concatenated to a vector in a high-dimensional space. Unfortunately, treating the image as a vector often leads to an extremely high-dimensional space, which often makes the evaluation of the covariance matrix very difficult and falls into the "black hole" of computational complexity. Two-dimensional PCA (2D PCA) [3] treats the image as a matrix directly and has been proved to be a row-based PCA mathematically [4]. However, the reason why PCA can perform only on the rows of images is not revealed yet.

In this paper, to solve the small sample size problem and confusions about 2D PCA, a novel framework called subspace evolution analysis is presented. In addition, the essence of 2D PCA and the connection between PCA and 2D PCA are investigated.

2. Subspace evolution analysis

2.1. The iterative sampling technique

To deal with the curse of dimensionality and small sample size problem invalidating the subspace analysis method, an iterative sampling technique is proposed. There are two kinds of the iterative sampling techniques in our algorithm. One is the iterative downsampling technique and the other is the iterative upsampling technique. Only the iterative downsampling technique is discussed in detail and the iterative upsampling technique can be defined in a reverse way.

Unlike the traditional downsampling technique, the iterative downsampling technique does not discard the rest information after the first sampling but samples the rest information iteratively until all the information is sampled. In such a way, the downsampling technique can decrease the dimensionality of the image and increase the training samples simultaneously without losing any information and increasing computational complexity. And the original information is scattered in several downsampled images.

Theorem 1. *The iterative sampling technique is an orthonormal transform. Using the iterative sampling technique to sample an image, the information is well retained and can be recovered completely.*

* Corresponding author. Tel.: +86 21 34202035; fax: +86 21 34202033.
E-mail address: zhouyue@sjtu.edu.cn (Y. Zhou).

0031-3203/$30.00 © 2006 Pattern Recognition Society. Published by Elsevier Ltd. All rights reserved.
doi:10.1016/j.patcog.2006.06.013

Proof. Define an identity matrix $E=[e_1, e_2, \ldots, e_m]$, and e_i is an $m*1$ column vector where the ith element is 1. v_ratio and h_ratio denote the sampling ratio along the vertical and horizontal direction, respectively.

Sample E with h_ratio gets $E_2 = [e_1, e_{1/h\_ratio+1}, \ldots, e_{i/h\_ratio+1}, \ldots, e_{n-1/h\_ratio+1}]$. Therefore, sampling image A with h_ratio can be obtained by

$$B = AE_2, \tag{1}$$

In generalization, sampling image A with v_ratio and h_ratio can be obtained by

$$B = E_1 A E_2, \tag{2}$$

where $E_1 = [e_1, e_{1/v\_ratio+1}, \ldots, e_{i/v\_ratio+1}, \ldots, e_{m-1/v\_ratio+1}]$.

According to the definition of the iterative sampling technique, sampling E with v_ratio and h_ratio, respectively, using the iterative downsampling technique can get

$$E_3 = [e_1, \ldots, e_{m-1/v\_ratio+1}, e_2, \ldots, e_{m-1/v\_ratio+2}, \ldots, e_{1/v\_ratio}, \ldots, e_m], \tag{3}$$

$$E_4 = [e_1, \ldots, e_{n-1/h\_ratio+1}, e_2, \ldots, e_{n-1/h\_ratio+2}, \ldots, e_{1/h\_ratio}, \ldots, e_n]. \tag{4}$$

Therefore, sample image A with v_ratio and h_ratio using the iterative downsampling technique can be obtained by

$$B = E_3 A E_4. \tag{5}$$

The image can also be recovered completely using the iterative upsampling technique by

$$A = E_3^{-1} B E_4^{-1} = E_3^T B E_4^T. \quad \square \tag{6}$$

2.2. Divide–conquer–merge strategy

According to Theorem 1, the image can be sampled with arbitrary sampling ratio and free from loss of information. Each downsampled image can be considered as a clone of original image with a bit variation. They are different from each other and can be treated separately. For the purpose of representing and recognizing the original image, the "divide–conquer–merge" strategy is adopted to incorporate subspace analysis technique and the iterative sampling technique.

Due to the length limit, only PCA is evaluated in our work, but other subspace analysis techniques, such as fisherfaces [2], can also be embedded in the framework. With sampling ratio decreasing, each downsampled image will inherit less and less information, so the subspaces will evolve from definite eigenfaces into ambiguous eigenfaces, as illustrated in Fig. 1. Therefore, we call the proposed method subspace evolution analysis. Table 1 shows the algorithm of subspace evolution analysis.

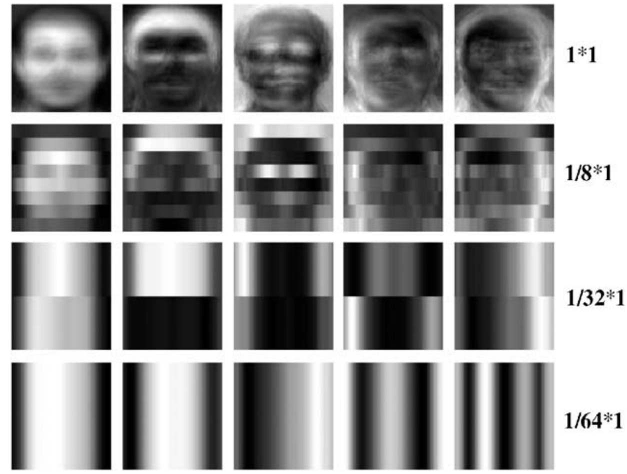

Fig. 1. The evolution of first eigenfaces.

Table 1
The algorithm of subspace evolution analysis

1. Divide stage:
   a) split the image into several downsampled images using the iterative downsampling technique.
2. Conquer stage:
   a) and perform subspace analysis method on the downsampled images;
   b) project the downsampled images into the leading eigenvectors separately and obtain the respective principal components; and
   c) reconstruct the downsampled images separately.
3. Merge stage:
   a) Synthesize the principal components which belong to the same image as the principal components of the original image.
   b) Reconstruct the original image by merging the reconstructed images of downsampled images using the iterative upsampling technique.

Theorem 2. *Subspace evolution analysis is the same as eigenface method in essence, which is able to guarantee that the information of the original image is well reserved.*

Proof. Given a set of images $A = \{A_k, 1 \leqslant k \leqslant M\}$, where M is the number of images and the image A_k with a size of $m*n$ is represented by $A_k = [\alpha_1^k, \alpha_2^k, \ldots, \alpha_m^k]^T$, α_i^k is ith row of kth image. First, A_k is split into several downsampled images using the iterative downsampling technique with $v\_ratio = 1/s$ and $h\_ratio = 1/t$. According to Eq. (5), the downsampled images can be represented by

$$B_k = \begin{pmatrix} B_{1,1}^k & \cdots & B_{1,t}^k \\ \vdots & \ddots & \vdots \\ B_{s,1}^k & \cdots & B_{s,t}^k \end{pmatrix} = E_3 A_k E_4, \tag{7}$$

where $B_{i,j}^k$ is a downsampled image and can be squeezed into a vector $x_{i,j}^k$. The covariance matrix can be obtained by

$$C = \frac{1}{M} \sum_{k=1}^{M} \sum_{i=1}^{s} \sum_{j=1}^{t} x_{i,j}^k (x_{i,j}^k)^T. \tag{8}$$