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Fast, robust and efficient 2D pattern recognition for re-assembling fragmented images☆

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Abstract

We discuss the realization of a fast, robust and accurate pattern matching algorithm for comparison of digital images implemented by discrete Circular Harmonic expansions based on sampling theory. The algorithm and its performance for re-assembling fragmented digital images are described in detail and illustrated by examples and data from the experimentation on an art fresco real problem. Because of the huge database of patterns and the large-scale dimension, the results of the experimentation are relevant to describe the power of discrimination and the efficiency of such method.
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1. Introduction

Since 1994, the authors have been involved in the fascinating attempt to recall to life a very important Italian art fresco (A. Mantegna, Cappella Ovetari, Chiesa degli Eremitani in Padova), fragmented in thousands of pieces by an Allied bombing in the Second World War (1944) [1–3]. Recently, a digital cataloging of the fragment images made possible to count their exact number (80735). The distribution of the areas shows that most are relatively small, with an average surface area of 5–6 cm^2, a total area of 77 m^2 versus an original surface of several hundreds square meters. These a priori data demonstrated the lack of continuous fragments for any given fragment and makes extremely improbable that any reconstruction will be successful using methods based on the outline shape of the fragments. There is no information on the possible location of the

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pieces on the huge original surface and it is unknown also the angle of rotation with respect to the original orientation. Some fairly good quality black and white photographs from between 1900 and 1920 exist, but they suffer from non-linear spectral distortion. A more detailed and complete description of the problem can be found in the contribution [4].

These facts impose that any feasible computer-based solution for a possible recomposition by comparison of the fragments and the fresco digital gray images must be

- *fast*, because of the huge number of fragments and original surface of the fresco;
- *robust*, because of the strong noise presence and intrinsic differences between the images, due to the damage of the bomb and the different photographic techniques;
- *accurate*, because of the fairly small dimensions of the fragments;
- *translation-rotation invariant*, because of the unknown original location and orientation of the fragments.

The request of a fast algorithm excludes the implementation of any comparison *pixel-by-pixel* and suggests that methods based on (compressed) series expansions can be more efficient. Besides other classical expansions, like Laguerre–Gauss [5] or Zernike polynomials [6] (fairly difficult to implement numerically), Circular Harmonic (CH) decompositions have found a relevant role in pattern matching because of their rotation invariance (self-steerable) properties and their effective and successful optical implementations [7–10]. In this paper, we want to present a digital/numerical implementation of compactly supported CHs and an effective 2D pattern recognition algorithm, based on these discrete expansions, which fulfills all the required properties listed above. The algorithm and its performance are described in detail and illustrated by examples and data from the experimentation on the fresco real problem. Because of the huge database of patterns and the large-scale dimension, the results of the experimentation are relevant to describe the power of discrimination and the efficiency of such method. Other problems can be interpreted in such a picture: experiments in character recognition, motion field detection and local rotation registration have also given very good results.

In literature, other kind of expansions have been presented as possible tools for pattern matching: To cite some, 2D (CH) wavelets [11–13] and multiscale self-steerable pyramid decompositions [14,15]. Even if they have given very interesting and promising results on small scale and local registration problems, it is still difficult to implement algorithms where a reasonable and feasible compromise among speed, robustness and location-rotation resolution can be realized on large scales.

The paper is organized as follows: Section 2 illustrates the CH expansions and their properties. In particular, it is shown that the moments constructed by correlation of a image with the CH system is a total information that can be used then for a complete comparison with an other signal. We discuss the discrete implementation of CH expansions by sampling and we will show that by limiting the system to a suitable and computable finite number of elements one can efficiently calculate the moments and preserve with optimal approximation *completeness*, *local orthonormality* and *self-steerability* also in the discrete domain. Section 3 illustrates the pattern recognition algorithm and its complexity is discussed with respect to a reference optimal method. In Section 4, numerical results and robustness of the algorithm in real cases are discussed and compared with the reference optimal method. An appendix collects notations and conventions used in the paper.

2. Discrete compactly supported Circular Harmonics

Compactly supported CHs arise as natural solutions of the Laplace eigenvalue problem on a disk under Dirichlet conditions [16], and they are related to relevant physical problems with rotation invariant symmetries. In fact, since the Laplacian commutes with rotations, CH are also eigenfunctions of any rotation operator. Let us introduce their formal definition as follows. We denote by $L^2(\Omega)$ the Lebesgue space of square-summable functions on $\Omega \subset \mathbb{R}^2$. Assume $\Omega_a \subset \mathbb{R}^2$ is a disk of radius $a > 0$. The system of CH functions on Ω_a is defined in polar coordinates by

$$e_{m,n,a}(r,\theta) = \frac{c_{m,n}}{a} J_m(j_{m,n} r/a) \mathrm{e}^{\mathrm{i}m\theta}, \quad m \in \mathbb{Z},\ n \in \mathbb{N}, \tag{1}$$

$$c_{m,n} = \pi^{-\frac{1}{2}} \left[\frac{\mathrm{d}J_m(s)}{\mathrm{d}s}\bigg|_{s=j_{m,n}} \right]^{-1}, \tag{2}$$

where J_m's are Bessel functions of the first kind of order $m \in \mathbb{Z}$, $(j_{m,n})_{n\in\mathbb{N}}$ is the sequence of their positive zeros [17], and $c_{m,n}$ is a normalization constant. We summarize their properties

(i) CH constitute an orthonormal basis for $L^2(\Omega_a)$ [16], i.e.,

$$\langle e_{m,n,a}, e_{m',n',a}\rangle := \int_{\Omega_a} e_{m,n,a}(x)\overline{e_{m',n',a}(x)}\,\mathrm{d}x = \delta_{(m,n),(m',n')},$$

being $\overline{f(x)}$ the complex conjugate of $f(x)$, and solve the Laplace eigenvalue problem:

$$e_{m,n,a} \in H_0^1(\Omega_a) \cap C^\infty(\overline{\Omega_a}),$$
$$\Delta e_{m,n,a} = -\left(\frac{j_{m,n}}{a}\right)^2 e_{m,n,a},$$
$$e_{m,n,a}(x) = 0 \quad \forall x \in \partial\Omega_a, \tag{3}$$

where $H_0^1(\Omega_a)$ is the Sobolev space of functions vanishing on the border $\partial\Omega_a$.

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