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Robust ellipse detection with Gaussian mixture models

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ABSTRACT

The Euclidian distance between Gaussian Mixtures has been shown to be robust to perform point set registration (Jian and Vemuri, 2011). We propose to extend this idea for robustly matching a family of shapes (ellipses). Optimisation is performed with an annealing strategy, and the search for occurrences is repeated several times to detect multiple instances of the shape of interest. We compare experimentally our approach to other state-of-the-art techniques on a benchmark database for ellipses, and demonstrate the good performance of our approach.

1. Introduction

Estimating the parameters of a shape is a problem that arises in different fields in computer vision. The problem is usually classified according to the parameters to be estimated, the knowledge or information about the shape and the type of observations that have been collected. Jian and Vemuri [1] proposed a method for estimating registration parameters between sets of points based on the $\mathcal{L}_2$ distance between density functions. Each data set is used for modelling a GMM. The transformation parameters between the two sets are computed by minimising the Euclidean distance ($\mathcal{L}_2$) between those two density functions. This metric has the advantage of having closed form solution when the density functions are Gaussian mixtures. We have explored this metric and the modelling of the GMM when estimating rigid transformation and for morphable model fitting [2–4]. We have shown how using non-isotropic Gaussians to represent shapes better can be beneficial for the robustness and accuracy of the results [5]. Following our previous work we propose here:

- To extend the framework based on $\mathcal{L}_2$ to estimate a parametric family of curves (i.e. ellipses).
- To propose a multidimensional modeling for the density functions in order to include additional information available with little impact on the computational efficiency of the approach.
- We propose a method for detecting multiple instances of an ellipse. This method is applied to ellipse detection in 2D point clouds and images. We evaluate the performance of our method with comparison to the state-of-the-art [6–10].

The remaining of the paper is structured as follows. In Section 2 we review the most relevant methods for estimating ellipses classified according to the strategy used. In Section 3 we propose our $\mathcal{L}_2$-based method for estimating the parameters of an ellipse. Section 5 extends our work to higher dimensional space allowing the integration of additional shape information. A second extension of our framework is presented in Section 4.2 for estimating multiple instances of ellipses. The experiments performed and results obtained are discussed in Section 6. Comparison with the state-of-the-art techniques is also provided in order to assess our proposed method. Finally, the conclusions of our work are presented in Section 8.

2. Literature review

Ellipse fitting is a challenging problem that arises in several fields. Some examples of applications are segmentation of cells [11], study of galaxies [12], medical diagnostics [13], camera calibration and face detection among others [14,15]. As many applications as there are of fitting ellipses there are also a great number of algorithms proposing solutions to this problem [16]. They are commonly classified in three categories: Least Square based methods, Hough Transform based methods and the most recent approach known as edge contour following methods.

2.1. Least Square based methods

Least Square based methods are usually classified into two categories according to the cost function to optimise. Those categories are (1) methods minimising a *geometric* error and (2) methods minimising an *algebraic* error [17–20]. The minimisation of the geometric error are

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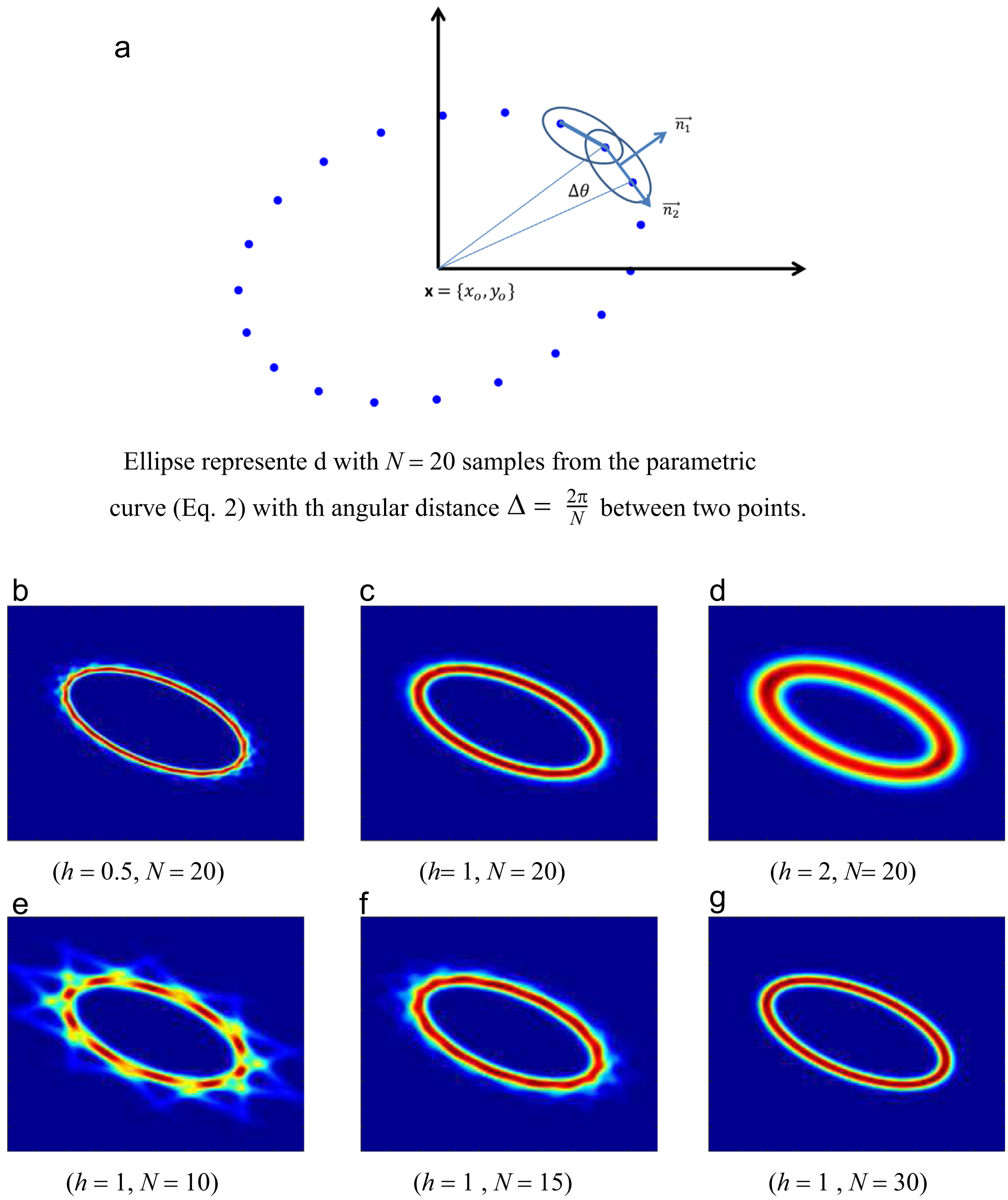

Fig. 1. Representation of an ellipse with GMMs computed using different values of orthogonal bandwidth h and various sampling rate N.

regarded as the most accurate methods for ellipse fitting and various computational schemes have been proposed [21]. However, those methods have several drawbacks. They need several iterations for solving the non-linear optimisation problem and they are very sensitive to noise. Therefore, the convergence of those methods is not guaranteed. Moreover, the convergence often depends on the accuracy of the initialisation.

Algebraic methods on the other hand are easier to implement and computationally efficient. However, the main problem of those methods is that they do no guarantee the result which will be an ellipse. A normalisation process is required in order to enforce the solution [22,23]. For instance, Szpak et al. [24] propose a penalty function that guaranteed an ellipse when using the Sampson distance. However, the result is then biased by the normalisation scheme chosen. Algebraic methods are less robust with respect to geometric methods when the data is coming from a small segment of an ellipse. Furthermore, accuracy of the results of those methods often depends on the initialisation. A method for finding a reasonable starting guess for those algorithms is proposed in [25].

2.2. Hough Transforms

The Hough Transform is a well known approach to detect ellipses [26–28]. It is based on a voting system where each edge pixel of an image is considered. This voting procedure is carried out in a parameter space, from which candidate ellipses are obtained as local maxima in an accumulator space that is explicitly constructed based on the parameters of the ellipse. Several algorithms have been proposed to improve performance of the HT method [29,9,30]. Some approaches explore the inclusion of additional information such as the directional property of the pixels [31,32]. Unfortunately, those methods are easily affected by possible noise in the image. A different strategy that improves the computational complexity of the HT is to sub-sample the data set. For instance, Kiryati et al. [33] used the Probabilistic Hough Transform (PHT) where just a portion of the edge pixel of an image