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Stable, fast computation of high-order Zernike moments using a recursive method

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ABSTRACT

Zernike moments and Zernike polynomials have been widely applied in the fields of image processing and pattern recognition. When high-order Zernike moments are computed, both computing speed and numerical accuracy become inferior. The main purpose of this study is to propose a stable, fast method for computing high-order Zernike moments. Based on the recursive formulas for computing Zernike radial polynomials, this study develops stable, fast algorithms to compute Zernike moments. Symmetry under group action and Farey sequence are both applied to shorten the computing time. The experimental results show that the proposed method took 5.292 seconds to compute the top 500-order Zernike moments of an image with 512×512 pixels. The normalized mean square error is 0.00124846 if 450-order moments are used to reconstruct the image. When computing the high-order Zernike moments, the proposed method outperformed other compared methods in terms of speed and accuracy.

1. Introduction

Zernike moments and Zernike polynomials have been widely applied in the fields of image processing and pattern recognition, such as shape matching [1,2], object recognition [3–5], image watermarking [6–8], image retrieval [9,10], signature authentication [11], and so on. Zernike moments are used in the various applications because the Zernike basis function satisfies the orthogonal property [12,13], implying that no redundant information overlapped between the moments [14]. Another advantage is that the magnitudes for Zernike moments are invariant to the rotation of the represented image, thereby enabling image matching and recognition to be performed under different orientations. Since the purpose of using Zernike moments is to expand an image into a series of coefficients with respect to orthogonal bases, the precision of pattern representation depends upon the number of moments used from the expansion. The low-order moments mainly represent the global contour of a pattern while the higher-order moments describe the detail. High-order moments are especially important for biometric studies because those human characteristics are subtle and complex patterns, requiring to represent as detailed as possible. For example, Haddadnia et al. [15] proposed a method for face recognition using a localization of facial information and high-order pseudo Zernike Moments as features and a radial basis function neural network as the classifier. Gayathri and Ramamoorthy [16] utilized high-order Zernike moments to verify palm prints. The reason for using high-order moments is that high-order Zernike moments can describe the detailed pattern content.

Computation of Zernike moments is considered a complex and lengthy process because the definition of Zernike polynomials involves factorial and trigonometric functions [17,18]. The product of those several functions results in a long number which takes a considerable time to compute. Moreover, the definition of each Zernike polynomial is independent of any other orders, though Zernike moments involve a hierarchical relationship. This means that each Zernike moment must be independently computed without taking any intermediate results from its lower-order moments. As a high-order moment is computed, its computation time greatly increases. For these reasons, two research issues—computing speed and numerical accuracy—arise when high-order Zernike moments are computed.

With regard to the issue of computing speed, the direct method, which follows the original definition to compute, takes an excessive amount of time to compute Zernike moments. To reduce computational complexity, Chong et al. [19] proposed the *q*-recursive method and performed a comparative analysis among several algorithms, including the direct method, Belkasim's [20], Prata's [21], and Kintner's methods [22]. Singh and Walia [18]

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modified Prata's method and presented a recursive relation to improve the computing speed. They suggested using the modified Prata's method for low orders (≤ 90), and Kintner's fast method for high orders (> 90) of Zernike moments. Qin et al. [23] introduced a recurrence relation to compute either a single Zernike moment or a whole set of Zernike moments. The fast computation method required fewer addition and multiplication operations and was executed in parallel. Shakibaei and Paramesran [24] presented a recursive formula to compute Zernike radial polynomials using a recursive relationship among radial polynomials. The derived recurrence relation is dependent on neither the degree nor the azimuthal order of the radial polynomials.

As regards numerical accuracy, the computational values in double-precision floating-point format become inaccurate when the order is greater than 45 in the original direct method. Papakostas et al. [17] analyzed the causes of computing errors in Zernike moments, performed an analysis of the finite precision error generation, and propagated the q-recursive method that computes the radial polynomials. In comparison, a recursive method can usually obtain more accurate values in high order than the direct method, though computing errors still exist in the higher order.

Previous studies have reported a tradeoff between computing speed and numerical accuracy. Furthermore, when high-order Zernike moments are computed, both computing speed and numerical accuracy become inferior. The main purpose of this study is not only to propose a recursive method for fast computation, but also an algorithm to yield accurate values of high-order Zernike moments.

2. Zernike moments and calculation methods

2.1. Zernike moments

The Zernike moments Z_{nm} can be regarded as the inner product of $f(x,y)$ with the basis function of Zernike polynomials $V_{nm}(x,y)$. The Zernike moments Z_{nm} are defined as

$$Z_{nm}=\frac{n+1}{\pi}\iint_{(x,y)\in D} f(x,y)V^{*}_{nm}(x,y)dxdy \tag{1}$$

where $V^{*}_{nm}(x,y)$ denotes the complex conjugation of $V_{nm}(x,y)$. For a complex number $z=x+iy$ (x,y are real numbers and $i=\sqrt{-1}$), the Zernike polynomial is given by

$$V_{nm}(z)=R_{nm}(r)e^{im\theta}=R_{nm}(r)(\cos(m\theta)+i\ \sin(m\theta)) \tag{2}$$

where $R_{nm}(r)$ is the Zernike radial polynomial, $r=|z|=\sqrt{x^2+y^2}$ is the vector length, the order n is a non-negative integer, the repetition m is an integer satisfying $n-|m|=$ an even number and $|m|\le n$, and θ is the angle between the vector and the x-axis counterclockwise. The radial polynomial $R_{nm}(r)$ is expressed as

$$R_{nm}(r)=\sum_{\substack{k=|m|\\ k-|m|\,:\,even}}^{n} R_{nmk}r^{k} \tag{3}$$

where

$$\begin{aligned}R_{nmk}&=(-1)^{\frac{n-k}{2}}\frac{\left(\frac{n+k}{2}\right)!}{\left(\frac{n-k}{2}\right)!\left(\frac{k+|m|}{2}\right)!\left(\frac{k-|m|}{2}\right)!}\\&=(-1)^{\frac{n-k}{2}}\begin{pmatrix}\frac{n+k}{2}\\ \frac{n-k}{2},\quad\frac{k+|m|}{2},\quad\frac{k-|m|}{2}\end{pmatrix}\\&=(-1)^{\frac{n-k}{2}}C_k^{\frac{n+k}{2}}C^k_{\frac{k-|m|}{2}}\end{aligned} \tag{4}$$

As a given $N\times N$ pixel image is projected into the unit disc D, the data of the image pixels can be regarded as a two-dimensional table $P(s,t)$ over the square $A:[-1/2,N-1/2]\times[-1/2,N-1/2]$. Let $A(Z)=A\cap\{(s,t)|s,t$ are integers$\}$, which represents all of the image pixels. The pixel (s,t) is projected via η onto the grid A_{st}, centered at $(x_s,y_t)=\eta(s,t)=\left(\frac{2s-N+1}{N\sqrt{2}},\frac{2t-N+1}{N\sqrt{2}}\right)$. This results in the corresponding function $f(x,y)$ over $A'=\eta(A)\subseteq D$ and $f(x,y)=f(\eta(s,t))=P(s,t)$. The discrete form of the Zernike moments in Eq. (1) is expressed as follows:

$$\hat{Z}_{nm}=\frac{2(n+1)}{\pi N^2}\sum_{(s,t)\in A(Z)}\sum P(s,t)R_{nm}(r)(cos(m\theta)-i\ \sin(m\theta)) \tag{5}$$

With the Zernike moments of an image $f(x,y)$, this image can be reconstructed by Eq. (6).

$$\begin{aligned}f(x,y)&=\lim_{M\to\infty}\sum_{n=0}^{M}\sum_{\substack{m=-n\\ m-n=even}}^{n} Z_{nm}V_{nm}(x,y)\\&=\lim_{M\to\infty}\sum_{n=0}^{M}(Z_{n0}R_{n0}(r)+2\sum_{\substack{m=0\\ m-n=even}}^{n} R_{nm}(r)(\mathrm{Re}(Z_{nm})cos(m\theta)\\&\quad-Im(Z_{nm})sin(m\theta)))\end{aligned} \tag{6}$$

where $Re(Z)$ and $\mathrm{Im}(Z)$ denote the real part and the imaginary part of a complex number Z, respectively. The reconstructed image function can be expressed as

$$\hat{f}(x,y)=\sum_{(n,m)\in I}\sum Z_{nm}V_{nm}(x,y) \tag{7}$$

where the indexed set $I=\{(n,m)|n\le M\}$ and M is an integer. As much higher orders of Zernike moments are used for reconstruction, this image content can be recovered much more completely. The original image and its reconstructed image can be compared with regard to the content for the evaluation of the computation of accurate Zernike moments. In comparison, the difference between the two images can be estimated by the normalized mean square error ($NMSE$), as expressed in Eq. (8). This measurement will be used to show the performance of stable computation among different methods.

$$NMSE=\frac{\iint_D|f(x,y)-\hat{f}(x,y)|^2dxdy}{\iint_D|f(x,y)|^2dxdy} \tag{8}$$

The subsequent Sections 2.2–2.4 present the three methods used to calculate the Zernike moments: the direct method, the q-recursive method, and Prata's method.

2.2. Direct method

The direct method follows the straightforward definition of radial polynomials and approximates the Zernike moments as in Eq. (5). The time complexity of the direct method is $O(M^3N^2)$, which is considered a high cost in terms of time complexity. In addition, when the high-order moments are calculated, the resulting moments become numerically instable and ina ccurate [18].

2.3. q-Recursive method

The q-recursive method, proposed by Chong et al. [19], uses the radial polynomials of higher repetition m (m is equal to the notation q in the q-recursive method) to derive the radial polynomials of the lower repetition m without involving any factorial terms. The q-recursive relation among radial polynomials is given by

$$R_{nm}(r)=K_1R_{n,m+4}(r)+\left(K_2+\frac{K_3}{r^2}\right)R_{n,m+2},\quad m=n-4,\ n-6,\ldots,1\ (\text{or }0) \tag{9}$$