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Tunable halfband-pair wavelet filter banks and application to multifocus image fusion ☆

Aryaz Baradarani *, Q.M. Jonathan Wu, Majid Ahmadi, Pankajkumar Mendapara

Department of Electrical and Computer Engineering, University of Windsor, Ontario, Canada

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ABSTRACT

Traditional maximally flat wavelet filters are highly regular but suffer from poor frequency-selectivity because of their wide transition band. In this paper, an efficient method is proposed for the design of biorthogonal perfect reconstruction wavelet filter banks, known as halfband-pair filter banks (HPFB), to be used in several applications in image processing and pattern recognition. The formulation is based on representation of a general halfband polynomial in the variable x. We first derive filter coefficients in the polynomial domain (in the variable x) in terms of the coefficients of the corresponding function in z-domain. Using convex optimization techniques, and due to the simple structure of a parametric polynomial in general, we can impose some free parameters to provide a tuning opportunity to optimize and control the wavelet filter characteristics. Perfect reconstruction and desired number of vanishing moments (NVM) are incorporated into the design procedure. The method is systematic, renders a reasonable optimization problem, and it offers wavelet filters ranging from the maximally flat to the sharpest transition band. Therefore, it can provide a useful design tool, with a fine-tuning option, which is required in many applications such as watermarking, detection, segmentation, fusion, denoising, and feature extraction. The application of the wavelet pairs, which have sharper transition band and better frequency-selectivity, is shown in multifocus imaging to obtain a fully focused image from a set of registered input images at varying foci by employing the distance transform and exponentially decaying function on the subbands in the wavelet domain. Various images are tested and experimental results compare favorably to the results in the literature.

1. Introduction

Availability of multisensor data and multiresolution analysis led to research opportunities to develop and improve many fields such as remote sensing, medical imaging, signal processing, machine vision, and military applications. Over the last two decades, multiresolution transformations had a large impact not only on applied math and engineering, but also on a variety of applications in several other subjects. Celebrated work of pioneers, e.g., Daubechies, Mallat, Vetterli, and many other outstanding techniques in wavelet design have served the entire community of science and technology. In recent years, multiscale representations of signals have been used in a number of image processing and computer vision applications including multifocus image fusion and shape-from-focus. Multiresolution analysis, or multiscale approximation, was basically referred to as the theory and design of discrete wavelet transform (DWT), and later on extended to multiwavelets, dual-tree complex wavelet transform (DT-ℂWT), higher-density discrete wavelet transform (HD-DWT), and framelets [6,20,27,32,33,37]. This family contains several desired properties such as orthogonality, biorthogonality, regularity, and continuity. Although these wavelets cannot be explicitly expressed, they can be written as a function of another wavelet in the family. In addition, there is a small group of wavelets in the literature known as crude wavelets for which the wavelet filters are generated using an explicit mathematical equation. Even with explicit, continuous, and theoretically infinite mathematical equations for these crude wavelets, still one needs to produce discrete finite filters for digital applications. The literature on pulse shaping and crude wavelets is vast and various pulses such as Gaussian, Mexican hat, hyperbolic, Meyer and Morlet which belong to the crude wavelet group have been investigated by researches for decades [14]. An interesting approach to edge detection in hyperbolic- and Gaussian-distributed pixel-intensity images using hyperbolic and Gaussian masks has been recently presented in [15]. In [16], hyperbolic kernels are further investigated and auto-term functions of the first-order

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* Corresponding author. Tel.: +1 519 992 7308; fax: +1 519 971 3695.
E-mail addresses: baradar@uwindsor.ca (A. Baradarani), jwu@uwindsor.ca (Q.M. Jonathan Wu), ahmadi@uwindsor.ca (M. Ahmadi), mendapa@uwindsor.ca (P. Mendapara).

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hyperbolic kernel, Choi-Williams (CW) kernel and nth-order hyperbolic kernel have been explicitly derived. Although many state-of-the-art applications take advantage of existing multiresolution transformations, the theory and design of new multiresolution systems is a difficult and challenging problem whose solution helps to improve relevant applications in image processing and pattern recognition. This paper deals with the design of a class of wavelet filter banks, halfband-pair filter bank (HPFB), which is referred to as tunable halfband pair (THP) throughout the text. The proposed design scheme offers wavelet filters ranging from the maximally flat (lowest frequency-selectivity) to the maximal passband/stopband width (sharpest transition band) along with a control and tuning opportunity over the filters characteristics required in multiresolution based applications.

Multiresolution analysis allows the decomposition of a signal into its approximation and details. On the theoretical level this is an analysis-synthesis configuration, while the practical view assumes that a signal is represented by its approximation coefficients at some scale and that it is decomposed in terms of its coefficients at a larger scale. As one must compute these coefficients on several different scales, the necessity of a wavelet filter bank is well understood. Two-channel orthogonal filter banks cannot have linear phase except for Haar wavelets. In contrast, biorthogonal filter banks have more degrees of freedom and allow linear phase, an important property that some special filters possess [33,37].

In its basic form a lowpass filter is *regular* if it has at least one zero at $z=-1$, the so-called number of vanishing moments (NVM). While regularity is an important task in wavelet-based compression, *frequency-selectivity* is also another significant factor to be considered within applications such as denoising, subband coding, classification and recognition. Traditional maximally flat wavelet filters [6] are highly regular but have poor frequency-selectivity. In [31], Rioul and Duhamel showed that regularity and frequency-selectivity are in an inverse relationship. The challenge here is to propose a wavelet design method that presents the best frequency-selectivity for the given regularity or degree of flatness. In other words, the problem is defined as how to design wavelet filters with the sharpest transition band for a fixed number of vanishing moments. This paper is motivated by the possibility, and the need for improvements of wavelet filters characteristics in two-channel perfect reconstruction filter banks. The necessity stems from the lack of a clear control over the frequency response of the filters in terms of passband/stopband edges, number of vanishing moments, and ripples. The possibility of such improvements is related to the flexibility and number of free parameters available in the design space, and the recent achievements in semidefinite programming (SDP) and convex optimization techniques in general [3,25].

Various approaches have been considered for the design of perfect reconstruction filter banks, e.g., the Remez exchange algorithm, least squares, and eigenfilter [27]. In [4], optimization of filter banks is investigated for invariant supervised texture segmentation. Multiscale directional filter bank is discussed extensively in [5] to suppress the aliasing effect, as well as to minimize the reduction in frequency resolution where the problem of aliasing in decimated bandpass images on directional decomposition has been addressed. Dumitrescu formulated an SDP problem [8] that improved the orthogonality error of our earlier orthogonal filter banks. In [9], and based on an SDP framework to guarantee the global optimality, it has been shown that an implicit form of regularity constraint imposition is much more appropriate in terms of numerical accuracy.

The idea of using parametric Bernstein polynomial (PBP) in wavelet design has been investigated by several researchers. In [41], a sum of squares based method was proposed for the design of halfband product filters for orthogonal wavelets. Zhang indicates that the well-known Remez exchange algorithm is an efficient approach for equiripple design of orthogonal FIR filters [39]. A generalized parametric quadrature mirror filter bank design technique was suggested in [10], where the authors introduced PBPs to approximate the orthogonal wavelet filters. Phoong et al. [27] proposed a design procedure for halfband-pair filter bank (HPFB) with the structural perfect reconstruction property, where the lowpass analysis filter, which is assumed to be a halfband filter, was used in a one-stage optimization process employing two kernels where the kernels are mostly assumed to be the same. Tay suggested a two-stage least squares design of HPFB, defining two different PBP-based kernels [35], where the objective function to be minimized to extract free parameters is the energy of ripples. Another method was proposed by Patil et al. [26] to design FIR wavelet filter banks using factorization of a halfband polynomial in the frequency domain. In [2], we have briefly shown that, using PBPs and similar to [35], one can setup a simple-to-implement SDP based formulation to obtain the optimal transition band for a given filter length and pre-specified NVM.

Motivated by the capability of parametric polynomials in filter design, and in the light of notable progress in optimization techniques and software packages, we first derive filter coefficients in the polynomial domain (in the variable x) in terms of the coefficients of the corresponding function in z-domain. We then define a new objective function and problem formulation based on SDP representation of the obtained polynomial. It is pointed out that due to use of SDP and the way we define the problem, for a fixed filter length and pre-specified filter characteristics, solution to this optimization problem can generate filter pairs with the maximal passband/stopband width. The proposed technique offers tuning opportunity on the passband and stopband widths, amplitude of ripples, and number of vanishing moments. Based on these factors, one can simply tune the specified requirements in an application.

An application of the wavelet filters designed in this paper is shown in multifocus imaging. The reconstruction of a geometric object and to retrieve spatial information from one or multiple observation is a challenging problem in computer vision. When a 3D scene is projected into a 2D image plane, depth information is lost. The object points on the focus plane appear sharp, and blurring is increased relatively with respect to the distance of imaging system to the focus plane. For a scene with considerable depth, points on the focus plane have a sharp appearance while the rest of the scene points are blurred. Taking the advantage of wavelet transform and employing filters with the sharpest transition band proposed in this paper, the transformation provides promising results to select the best fitting feature pixel among the input images. While the accuracy of pixel information highly depends on its frequency content in wavelet subbands, it is reasonable to use wavelets with the sharpest transition band width. This idea motivated us to use these wavelets for multifocus imaging and depth map estimation. Experimental results and quantitative comparisons show that the proposed framework improves the results obtained utilizing traditional methods of fused image extraction via multifocus imaging. In addition to the filters designed in this paper, the performance of the algorithm is investigated employing the Daubechies orthogonal, Cohen–Daubechies–Feauveau biorthogonal, and the crude wavelets family.

The rest of paper is organized as follows. In Section 2, we show that a general halfband function in the z-domain can be derived in terms of its coefficients in the polynomial domain in the variable x, followed by the problem formulation based on SDP representation of the obtained polynomials. This is shown in Section 3. Design examples and discussions are presented in Section 4. In Section 5, we propose a technique for multifocus imaging and depth map