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Fuzzy complex numbers and their application for classifiers performance evaluation

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ABSTRACT

There are a variety of measures to describe classification performance with respect to different criteria and they are often represented by numerical values. Psychologists have commented that human beings can only reasonably manage to process seven or-so items of information at any one time. Hence, selecting the best classifier amongst a number of alternatives whose performances are represented by similar numerical values is a difficult problem faced by end users. To alleviate such difficulty, this paper presents a new method of linguistic evaluation of classifiers performance. In particular, an innovative notion of fuzzy complex numbers (FCNs) is developed in an effort to represent and aggregate different evaluation measures conjunctively without necessarily integrating them. Such an approach well maintains the underlying semantics of different evaluation measures, thereby ensuring that the resulting ranking scores are readily interpretable and the inference easily explainable. The utility and applicability of this research are illustrated by means of an experiment which evaluates the performance of 16 classifiers using different benchmark datasets. The effectiveness of the proposed approach is compared to conventional statistical approach. Experimental results show that the FCN-based performance evaluation provides an intuitively reliable and consistent means in assisting end users to make informed choices of available classifiers.

1. Introduction

Pattern classification has been successfully applied to many application domains. For instance, classifiers have been developed in conjunction with feature selection approaches [1–3] to perform tasks such as image analysis [4], face recognition, and remote sensing. However, classifiers which are applied to different problem domains and trained by various learning algorithms can perform quite differently. In fact, evaluating classifier performance is perhaps one of the most deceiving and tricky problems in classifier design [5]. To tackle this important problem, a variety of measures have been proposed to describe the classification performance with respect to different criteria, ranging from classification accuracy and error rate, through storage complexity and computation time to sensitivity and robustness [6–8].

In principle, performance measures can be qualitative or quantitative. Quantitative measures are naturally expressed by numerical values. However, using such seemingly precise measures to compare a number of classifiers, their performances may turn out to be very close in value. Such pure numerical values with small differences may not make much sense to the user who would like to make an informed choice of available classifiers. It would be more appropriate and often desirable to describe the relative performance of the classifiers using linguistic terms, such as *good*, *average* and *bad*. The assessment in qualitative measures often reflects the knowledge of domain experts and such measures are usefully represented by linguistic terms. Compared to numerical values, linguistic terms make it easier for users to understand the evaluation outcome. Indeed, human beings appear to use qualitative reasoning when initially attempting to gain an understanding of a problem.

It is worth noting that in order to obtain a fair evaluation of classification performance, several measures may need to be taken into account concurrently. For example, *precision* and *recall* are two widely used statistical measures which jointly provide a common indication of classifier performance. However, for many classification tasks, these two statistical measures should not be utilised in isolation, as neither measure alone contains sufficient information to assess the performance. It can be trivial to achieve a *recall* score of 1.0 by simply assigning all instances to a certain class. Similarly, *precision* may remain high by classifying only a few instances. To combat this, *precision* and *recall* are usually combined into a single measure, such as the *F-measure* which is the weighted harmonic mean of these two measures [9]. Unfortunately, in so doing, the underlying semantics associated with these two base measures may be destroyed, even if a qualitative

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version of the *precision* and *recall* measures are used. Thus, it is of great interest and potentially beneficial to establish a new mechanism which can maintain the associated semantics when performing evaluation without necessarily using just one transformed measure. Inspired by this observation, this paper proposes a novel framework of fuzzy complex numbers (FCNs) that will entail effective and efficient representation of different types of evaluation measures concurrently and explicitly.

Note that the term FCN is not new; the concept of complex numbers has been proposed in the literature. For example, a form of *fuzzy complex numbers* has been defined in [10] as a mapping from the conventional complex number plane to the real-valued interval [0,1]. Such an FCN is therefore, simply a type-1 fuzzy set [11]. Work on the differentiation and integration of this type of FCNs has been proposed in [12,13], with more advanced follow-on research on their mathematical properties reported in [14–17]. Recently, in combining fuzzy complex analysis and statistical learning theory, important theorems (of a learning process) based on fuzzy complex random samples were developed [18]. This work further demonstrates the interesting properties of the so-called *rectangular fuzzy complex numbers*, which are a special type of FCN as proposed in [10]. Another relevant development is the notion that relates real complex numbers to fuzzy sets [19]. It introduces a new type of set, named *complex fuzzy sets*, to allow the membership value of a standard fuzzy set to be represented using a classical complex number. However, as discussed in [19], it may be difficult to identify suitable real-world problems for the use of such complex-valued memberships. Despite this obstacle, work has continued along this theme of research. This is evident in that *complex fuzzy sets* have been integrated with propositional logic to construct a specific instance of fuzzy reasoning systems [20].

Existing research regarding the concept of FCNs is all framed by either giving conventional complex numbers a real-valued membership or assigning a fuzzy set element to a complex number as its membership value. These approaches are rather different from what is proposed in this paper, where both the real and imaginary values of an FCN are in general, themselves fuzzy numbers; each with an embedded semantic meaning. By extending the initial definition and calculus of the proposed FCNs as given in [21], important algebraic properties, including closure, associativity, commutativity and distributivity of such FCN are established in the present work. This helps to support the aggregation process of FCNs. This new aggregation approach enhances the original work of [21] by allowing an arbitrary number of components of an FCN to be integrated in a random order. Further, the newly derived modulus of this type of FCN is introduced to impose an order over a given set of FCNs. Apart from these theoretical contributions, this work is applied to a completely new problem domain to gauge the performance of classifiers. This differs significantly from what is reported in [21]. The underlying development of this new approach to FCNs is general. It offers great potential for other application problems which exhibit similar characteristics as those of multi-criteria performance evaluation (e.g. student performance evaluation [22]).

The rest of this paper is organised as follows. Section 2 proposes the novel approach to the notion of FCNs, which extends real-valued complex numbers to representing two-dimensional linguistic-valued measures concurrently. In Section 3, this approach is utilised to construct a general linguistic evaluation method which effectively ranks the overall performance of different classifiers. For computational simplicity, such a general evaluation method is specified using the linear triangular fuzzy sets. Details of the implemented classifier evaluator are also presented in this section. Section 4 describes the experimentation carried out on standard benchmark datasets and discusses the evaluation results. The paper is concluded in Section 5, with the perspective of further work pointed out.

2. Fuzzy complex numbers

2.1. Prerequisites

2.1.1. Fuzzy numbers

Fuzzy numbers are a special type of fuzzy sets which can be used to represent imprecise quantities such as *about 0.6*. Fuzzy numbers map real values from $\mathbb{R}$ on to a closed interval [0,1].

Definition 1. (Fuzzy numbers [23]) A fuzzy number, $\tilde{a}$, is defined as

$$\tilde{a} = \{(x,\mu_{\tilde{a}}(x)) | \mu_{\tilde{a}}(x) \in [0,1], x \in \mathbb{R}\},$$

and satisfies the following properties:

(a) Continuity: $\mu_{\tilde{a}}(x)$ is a continuous function mapping from $\mathbb{R}$ to a closed interval [0,1].
(b) Normality: i.e. $\exists x \in \mathbb{R}$ and $\mu_{\tilde{a}}(x) = 1$.
(c) Convexity: i.e. $\forall x,y,z \in \mathbb{R}$, if $x \le y \le z$ then $\mu_{\tilde{a}}(y) \ge \min(\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(z))$.
(d) Boundness of support: i.e. $\exists S \in \mathbb{R}$ and $\forall x \in \mathbb{R}$, if $|x| \ge S$ then $\mu_{\tilde{a}}(x) = 0$.

2.1.2. Extension principle

The extension principle [24] provides a fundamental mechanism to translate conventional boolean set-based concepts into their fuzzy-set counterparts. In this work, it forms the foundation to derive the arithmetic operations of the proposed FCNs.

Definition 2. Let f: $\mathbb{R}^n \longrightarrow \mathbb{R}$ be a function and $A_1, \ldots, A_n$ be fuzzy sets. Then $B = f(A_1, \ldots, A_n)$ is a fuzzy set with the following membership function:

$$\mu_B(y) = \bigvee_{y = f(x_1,\ldots,x_n)} (\mu_{A_1}(x_1) \wedge \cdots \wedge \mu_{A_n}(x_n)). \tag{1}$$

Note that the operators $\wedge$ and $\vee$ above denote a given t-norm and s-norm, respectively. Throughout this paper, they are interpreted using the min and max operators.

2.2. Definition of FCNs

Inherit from the real complex numbers, an FCN, $\tilde{z}$, is defined in the form of

$$\tilde{z} = \tilde{a} + i\tilde{b}, \tag{2}$$

where both $\tilde{a}$ and $\tilde{b}$ are fuzzy numbers with membership functions $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$, regarding a given domain variable x. $\tilde{a}$ is the real part of $\tilde{z}$ while $\tilde{b}$ represents the imaginary part, i.e. $Re(\tilde{z}) = \tilde{a}$ and $Im(\tilde{z}) = \tilde{b}$.

An FCN can be visually shown as in Fig. 1. Importantly, in general, for a given $\tilde{z}$, both $Re(\tilde{z})$ and $Im(\tilde{z})$ are fuzzy. If $\tilde{b}$ does not exist, $\tilde{z}$ degenerates to a fuzzy number. Further, if $\tilde{b}$ does not exist and $\tilde{a}$ itself degenerates to a real number, then $\tilde{z}$ degenerates to a real number.

2.3. Operations on FCNs

The operations on the proposed FCNs are a straightforward extension of those on real complex numbers. Let $\tilde{z}_1 = \tilde{a} + i\tilde{b}$ and $\tilde{z}_2 = \tilde{c} + i\tilde{d}$ be two FCNs, where $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ and $\tilde{d}$ are fuzzy numbers with