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Identifying centers of circulating and spiraling vector field patterns and its applications

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ABSTRACT

Identification of centers of circulating and spiraling vector fields, sources and sinks are important in many applications. Tropical cyclone tracking, rotating object identification, analysis of motion video and movement of fluids are but some examples. In this paper, we introduce a method for finding the centers of circulating and spiraling vector field patterns. It can handle vector fields with multiple centers and is robust against noise. We provide a theoretical analysis on the validity of our method, and application examples in the fields of multimedia processing and meteorological computing to demonstrate its practical use.

1. Introduction

Spiral, circular, or elliptical 2D vector fields, as well as sources and sinks, are encountered in many applications. For example, in multimedia, circular and elliptical motion fields are created by motion compensated prediction [1] of rotating objects [2] or wipe scene changes [3] in video sequences. In meteorology, motion vector fields constructed from satellite or radar images [4–6] show circulating or spiraling structures of tropical cyclones (TCs), pressure systems and tornadoes [7]. Rotational vector fields are studied in developing rotating components of electrical appliances such as hair dryers, washing machines, and fans in industry. Orientation fields which show circulating or spiraling patterns also draw attention to computer vision researchers [8–11]. Location of the centers of these 2D vector fields is important, as it provides useful information of the field structure. In [2], centers of rotating objects in video sequences are found to help object segmentation and tracking. To meteorologists, these 2D vector fields are analyzed to study cloud or radar echo movements [12,13], locate centers of TCs [7,14–16] and tornadoes. Additional applications also arise in aerodynamics [17], fluid mechanics, and medical imaging.

Straightforward as it may seem, locating the centers of rotating fields is practically challenging as real-life vector fields are seldom perfect. They are often incomplete, distorted, or noisy. Robust methods tolerant to noise are thus needed. In this paper, we introduce a practical and flexible method for finding centers of circulating and spiraling vector fields. It not only allows handling of vector fields with multiple centers, it is also robust against noise. We compare our method with common vector field pattern matching and algebraic analysis approaches, and give suggestions on the choice of parameters.

The paper is organized as follows. We first review the existing methods in the literature in Section 2. In Section 3, a mathematical model of circulating and spiraling vector field patterns and the method for identifying their centers are introduced. The experimental setup for evaluating the efficiency and effectiveness of the proposed method against common approaches is then described in Section 4. In Sections 5 and 6, we show the soundness of our algorithm and its robustness against noise using experiments on synthetic vector fields. Efficiency issues are then discussed in Section 7. The practicality of the method in handling fields with different scales and noises is demonstrated in Sections 8 and 9 using examples in multimedia video processing and meteorological computing. The effect of the parameters, and the limitations of the algorithm are discussed in Section 10, where we also propose methods to handle these limitations. Finally, a summary in Section 11 concludes the paper.

2. Related work

The simplest method to identify the centers of a circulating or swirling vector field $\mathbf{F}$ is to locate regions with high magnitude of vorticity, $\nabla \times \mathbf{F}$, which represents the amount of local rotation. To identify sources or sinks, the divergence, $\nabla \cdot \mathbf{F}$, can be used. An adaptation of the method includes circulation analysis, in which the vorticity of an area is found by the equation $\int\int_R (\nabla \times \mathbf{F} \cdot \mathbf{k})\, dx\, dy = \oint_C \mathbf{F} \cdot d\mathbf{r}$, where R is a closed bounded region on the x–y plane whose boundary is C. However, such simplistic methods are ineffective on incomplete or noisy fields.

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Table 1
Summary of related work.

Vector operations		Idea: Given a vector field **F**, the location with high magnitude of vorticity, $\nabla \times \mathbf{F}$, is the center of circulating or swirling fields. Location of sources or sinks can be identified by the divergence, $\nabla \cdot \mathbf{F}$ *Circulation analysis*: Find regions R bounded by C with high values of $\int\int_R (\nabla \times \mathbf{F} \cdot \mathbf{k})\,dx\,dy = \oint_C \mathbf{F} \cdot d\mathbf{r}$
Vector field pattern matching	Reference	Idea: The center is the location that best fits a template from the input vector field under a similarity measure
	[18]	*Correlation* between the defined templates (Center, Saddle, and Node) and the vector field is done to find the location with a similarity score that exceeds a predefined threshold
	[20]	*Convolution* between the defined templates (left- and right-handed swirl flows, converging flow, diverging flow, vortices, and parallel flows) and the vector field is done to create a tensor field. The location with the largest eigenvalue is the answer
	[19]	*Clifford convolution* is done to find the orientation of the defined templates, followed by scalar correlation on the rotated mask to locate the maxima
Algebraic analysis	Reference	Idea: According to the dynamical system properties of the fields, singularities are identified. The strongest singularities that is classified as "center" is regarded as the answer
	[21]	Vector field is decomposed into irrotational and solenoidal components through *discrete Helmholtz–Hodge Decomposition*. The extrema of the corresponding potential function are the locations of vortices, sources or sinks
	[22,23]	Vector field is decomposed into irrotational and solenoidal components through *2D Fourier transform*. The extrema of the corresponding squared potential and stream function are the locations of critical points, which are then classified by checking the eigenvalues of the Jacobian matrix
	[8]	An *isotangent-based algorithm* is used to estimate flow parameters for critical point classification
	[9,10]	*Linear estimator* is used to estimate the phase portraits. Curl, divergence and deformation are used for critical point classification
	[24]	Flow orientation, strength and phase portrait characteristics are *combined* to form a measure for critical point identification
Structural analysis	Reference	Idea: A voting technique is employed. The center of the flow is the point covered by the largest number of sectors generated by rotated vectors of the motion vector field
	[2]	Each vector is rotated by $\pi/2$ with a sector span to *offset the rounding errors of motion estimation algorithms*. The method is designed to handle a single circular field
	[25]	Each vector is rotated by ω with a sector span σ to *handle vector fields under orthographic projection*

Previous work that address the issue mainly solve the problem using two approaches: (1) vector field pattern matching, and (2) examination of dynamical system properties of vector fields using algebraic analysis. A summary of previous work on center identification from vector fields is given in Table 1.

The idea of vector field pattern matching algorithms is to take the center as the location of the input vector field that best fits a template under some similarity measure, such as sine metric [18] or convolution [19,20].

In [18], three 2D templates, "Center", "Saddle" and "Node" are defined. The similarity measure used is an absolute sine metric. The location with a similarity score that exceeds a predefined threshold is regarded as the answer. A method for doing quality check of the answers is also proposed in the paper.

As another example, in [20], a set of templates, including left- and right-handed swirl flows, converging flow, diverging flow, vortices, and parallel flows, are defined for 3D flow characterization. These templates and their rotated counterparts are convolved with the vector field to obtain a similarity score. The linear combination of the template similarity scores creates a tensor field, which describes the local properties of the vector field. The location with the largest eigenvalue represents the output, and the corresponding eigenvector indicates the symmetry axis of the structure.

Another similar approach for 3D flow characterization is proposed in [19], where Clifford convolution is first used to find the orientation of the templates. The scalar correlation between the rotated templates and the field is then calculated to locate the maxima, which are taken as answers.

Methods employing vector field pattern matching approach are flexible as different templates can be defined for finding different flow patterns. However, the size and the pattern of templates have to be similar to the patterns in the vector field.

Another approach for finding centers of rotating vector fields is to examine the dynamical system properties of the fields by algebraic analysis. First, the phase portrait of the vector field is computed. Next, critical points are located and classified. Besides centers, vector field singularities such as swirls, vortices, sources and sinks can also be identified.

To estimate the flow parameters for phase portrait computation, the vector field can first be decomposed into solenoidal and irrotational components through discrete Helmholtz–Hodge Decomposition [21] or 2D Fourier transformation [22,23]. From these components, the corresponding potential functions are derived, and the local extrema are the locations of critical points. Refs. [22,23] further filtered the less significant singularities by an iterative algorithm with Bhattacharyya distance and Rankine model as the constraints. The singularities in the filtered answer set are then classified according to the eigenvalues of the Jacobian matrix with respect to the position of a critical point.

Another set of work based on phase portrait of the flow is developed by Rao and Jain [8–11]. The authors proposed an isotangent-based algorithm in [8] to locate critical points on oriented textures. An isotangent line, which is a straight line proven to pass through a critical point [8], is fitted to each point with the same flow orientation. Least median of squares estimators are then applied to find the point that is closest to all the isotangent lines, and reject those lines that are outliers. The accepted isotangent lines are then used to estimate the parameter set for phase portrait classification of the identified critical point.

Another weighted least square error estimator is proposed in [9,10] for the estimation of phase portraits. The local properties of the vector field, including curl, divergence, and deformation are checked for flow pattern classification. These methods are used in applications such as semiconductor wafer inspection, and lumber defect characterization [11].

An alternative way for dynamical system estimation is proposed in [24], in which the flow field orientation, strength, and phase portrait characteristics are combined to form a measure for critical point identification. Potential critical points are then accepted or rejected by estimating the local flow field characteristics, such as isocline properties [24].