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Aggregation distance measure and its induced similarity measure between intuitionistic fuzzy sets ☆

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ABSTRACT

Various distance and similarity measures in Atanassov's intuitionistic fuzzy sets (IFSs) are applied to practical problems. However, it is difficult to decide which one is the most suitable for measuring the degree of distance or similarity, especially in the case of the occurrence of conflict results in practice. In this paper, we propose the concept of aggregation distance measure for IFSs based on aggregation functions without zero divisors to help decision makers to make final decisions. Furthermore, a novel technique is introduced to construct similarity measures from a given distance measure with respect to non-filling fuzzy negations, which gives a new direction for the construction method for similarity measures of IFSs. Some illustrative examples in applications such as pattern recognition, fuzzy clustering and decision making are used to investigate the effectiveness of the proposed measures.

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1. Introduction

Since the seminal work of Zadeh, the fuzzy set (FS) theory characterized by a membership function between zero and one has become a useful tool to handle with imprecision and uncertainty [1]. In real-life situations, taking the hesitation or uncertainty about the membership degree into consideration, the degree of non-membership is not always equal to one minus the degree of membership, which is treated as reasonable in FS theory. To address this issue, Atanassov introduced the notion of intuitionistic fuzzy sets (IFSs) as an extension of FS, in which not only the degree of membership is given, but also the degree of non-membership degree [2].

Ever since IFSs' appearance, many authors have paid great attention to the measures of distance and similarity between IFSs. Szmidt and Kacprzyk proposed four distance measures between IFSs, which were in some extent based on the geometric interpretation of intuitionistic fuzzy sets [3]. Li and Cheng proposed similarity measures of IFSs based on an axiomatic approach and applied these measures to pattern recognition [4]. But it was later pointed out that Li and Cheng's measures are not always effective in some cases [5–7]. Hung and Yang proposed several similarity measures of IFSs based on Hausdorff distance and $\mathbf{L}_p$ metric which can effectively be used with linguistic variables [8,9]. Hatzimichailidis et al. introduced distance metric between IFSs which makes use of matrix norms and fuzzy implications [10]. Farhadinia presented a new similarity measure for IFSs by using the convex combination of endpoints and also focusing on the property of *min* and *max* operators [11]. Iancu also introduced two families of similarity measures based on Frank t-norms [12]. Most of these aforementioned measures in different formats are originated from axiomatic definitions. Recently, Baccour et al. and Xu and Chen gave a comprehensive overview of distance and similarity measures of IFSs [13,14].

Measuring the distance and similarity between IFSs is now being extensively applied in many research fields, such as pattern recognition, fuzzy clustering and decision making. However, there may exist inconsistent results if we adopt different distance and/or similarity measures in practical applications as exemplified in Section 4, which will certainly get the decision makers into trouble. This situation can arise in a decision-making problem. How to make a decision to choose the optimal alternative from the conflict conclusions is still a problem to be solved. Li et al. gave a comparative analysis of the existing similarity measures for IFSs to benefit selection of similarity measures [15].

In this paper we attempt to deal with this problem from another viewpoint. Taken n distances we interested in as inputs, it is expected to produce a reasonable output, based on which the final decision is made. It is within the domain of the theory of aggregation functions. In this paper, the distance measures are aggregated as an aggregation distance by using aggregation functions. The weighted average of the existing distance measures is accepted as the overall evaluation. We propose two approaches to set the weights based on mathematical

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foundations. (I) We select the optimal solution of the shortest distance between a moving point and a vector with all its components degrees of distance we care about as the expectation. (II) We choose the normalized associated eigenvector of spectral radius of a non-negative symmetric matrix as an assignment and the weighted average of the degrees of distance the aggregation distance. Moreover, similarity measures generated by non-filling fuzzy negations for a given distance measure are provided to meet the axiomatic definition.

The rest of this paper is organized as follows: In Section 2, we recall basic concepts of intuitionistic fuzzy sets and some commonly used distance and similarity measures between IFSs. In Section 3, the aggregation distance measure for IFSs aggregated by an aggregation function without zero divisors is provided, as well as its induced similarity measure generated from a given distance measure with respect to a non-filling fuzzy negation. And their applications in pattern recognition, fuzzy clustering and decision making are given in Section 4. Finally, Section 5 concludes the present paper.

2. Preliminaries

In this section, we briefly recall some basic concepts relating to IFSs and some popular distance and similarity measures between IFSs.

Definition 2.1 ([2]). Let a (crisp) set E be fixed. An Atanassov's intuitionistic fuzzy set A in E is an object of the form $A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle | x \in E\}$, where functions $\mu_A(x), \upsilon_A(x) : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$ to A, respectively, and for every $x \in E$, $0 \leqslant \mu_A(x) + \upsilon_A(x) \leqslant 1$.

The function $\pi_A(x) : E \to [0,1]$, given by $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ defines the intuitionistic index of the element x in set A [2,16]. Specially, if $\pi_A(x) = 0$ for each $x \in E$, then the IFS A degenerates into a fuzzy set.

Let A and B be two IFSs given as $A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle | x \in E\}$ and $B = \{\langle x, \mu_B(x), \upsilon_B(x)\rangle | x \in E\}$. Denote $A^c = \{\langle x, \upsilon_A(x), \mu_A(x)\rangle | x \in E\}$;

$A \subseteq B \Leftrightarrow (\forall x \in E)(\mu_A(x) \leqslant \mu_B(x) \& \upsilon_A(x) \geqslant \upsilon_B(x))$;

$A = B \Leftrightarrow (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \upsilon_A(x) = \upsilon_B(x))$;

$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \upsilon_A(x) \wedge \upsilon_B(x)\rangle | x \in E\}$;

$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \upsilon_A(x) \vee \upsilon_B(x)\rangle | x \in E\}$.

Let IFS(E) denote the family of all IFSs in the universe E.

Definition 2.2 ([17]). Let d be a mapping $d : \mathrm{IFS}(E) \times \mathrm{IFS}(E) \to [0,1]$. If $d(A, B)$ satisfies the following properties:

(DP1) $0 \leqslant d(A, B) \leqslant 1$;
(DP2) $d(A, B) = 0$ if and only if $A = B$;
(DP3) $d(A, B) = d(B, A)$;
(DP4) If $A \subseteq B \subseteq C$, then $d(A, C) \geqslant d(A, B) \vee d(B, C)$.

Then $d(A, B)$ is a distance measure between A and B.

Definition 2.3 ([4,7]). Let s be a mapping $s : \mathrm{IFS}(E) \times \mathrm{IFS}(E) \to [0,1]$. $s(A, B)$ is said to be the degree of similarity between A and B if $s(A, B)$ satisfies the following properties:

(SP1) $0 \leqslant s(A, B) \leqslant 1$;
(SP2) $s(A, B) = 1$ if and only if $A = B$;
(SP3) $s(A, B) = s(B, A)$;
(SP4) If $A \subseteq B \subseteq C$, then $s(A, C) \leqslant s(A, B) \wedge s(B, C)$.

It is proved that if $d(A, B)$ is a distance measure between IFSs A and B, then $s(A, B) = 1 - d(A, B)$ is a similarity measure of A and B [17–19].

Remark 2.4. There are other ways to define axioms of distance and similarity measures. For example, Hung and Yang [20] gave an axiomatic definition of the distance and similarity measure for IFSs extended from FSs directly. Li et al. [15] suggested similarity measure between IFSs should also satisfy

(SP5) $S(A, A^c) = 0$ if A is a crisp set

to make the definition of similarity measure more strict and precise.

Several kinds of distance measures have been investigated in the literature. Here, we recall the widely used ones.

Let $E = \{x_1, x_2, \ldots, x_n\}$ be a discrete set of universe, A and B two IFSs in IFS(E). Wang and Xin [17] proposed a distance measure between A and B as follows:

$$d^p(A, B) = \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} [\varphi_\mu(x_i) + \varphi_\upsilon(x_i)]^p}, \tag{1}$$

where $\varphi_\mu(x_i) = \frac{|\mu_A(x_i) - \mu_B(x_i)|}{2}$, $\varphi_\upsilon(x_i) = \frac{|\upsilon_A(x_i) - \upsilon_B(x_i)|}{2}$ and p is a positive integer.

Hung and Yang [8] proposed several similarity measures of IFSs based on Hausdorff distance. The Hausdorff distance $d_H(A, B)$ between A and B is defined as:

$$d_H(A, B) = \frac{1}{n} \sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\upsilon_A(x_i) - \upsilon_B(x_i)|\}. \tag{2}$$

Based on this line of research, Hung and Yang [9] adopted the concept of $\mathbf{L}_p$ metric to define the distance between A and B: $(p \geqslant 1)$

$$L_p(A, B) = \frac{1}{n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)|^p + |\upsilon_A(x_i) - \upsilon_B(x_i)|^p)^{\frac{1}{p}}. \tag{3}$$

Note that $d_H(A, B)$ is also called Hamming distance by Grzegorzewski [21]. And we also have $\lim_{p \to +\infty} L_p(A, B) = d_H(A, B)$. It should be noticed that $0 \leqslant L_p(A, B) \leqslant 2^{\frac{1}{p}}$. For this reason, it can be normalized as

$$L_p^{\mathrm{mod}}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{2}|\mu_A(x_i) - \mu_B(x_i)|^p + \frac{1}{2}|\upsilon_A(x_i) - \upsilon_B(x_i)|^p\right)^{\frac{1}{p}} \tag{4}$$

to satisfy (DP1). Specially, it follows that $L_1^{\mathrm{mod}}(A, B) = d^1(A, B)$ (also called the normalized Hamming distance by Burillo and Bustince [22]) and $\lim_{p \to \infty} L_p^{\mathrm{mod}}(A, B) = d_H(A, B)$.

Grzegorzewski [21] also introduced the Euclidean distance between IFSs A and B as follows:

$$d_E(A, B) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} [\max\{(\mu_A(x) - \mu_B(x))^2, (\upsilon_A(x) - \upsilon_B(x))^2\}]}. \tag{5}$$

3. Aggregation distance measure and its induced similarity measure of intuitionistic fuzzy sets

In this section, the aggregation distance measure of IFSs on the basis of the existing distance measures is presented, so are the similarity measures generated from a given distance measure with respect to non-filling fuzzy negations.

Definition 3.1 ([23]). Let a mapping $f : [0,1]^n \to [0,1]$ $(n > 1)$ satisfy the following properties.

(P1) f is idempotent at $(0, 0, \ldots, 0)$ and $(1, 1, \ldots, 1)$, i.e., $f(0, 0, \ldots, 0) = 0$ and $f(1, 1, \ldots, 1) = 1$;
(P2) f is monotonic increasing in each of its components, i.e., if $x_i \leqslant y_i$, $i \in \{1, 2, \ldots, n\}$, then $f(x_1, x_2, \ldots, x_n) \leqslant f(y_1, y_2, \ldots, y_n)$.

Then f is referred to as an n-ary aggregation function.

Let f be an aggregation function. f is said to have zero divisors if there exists $x_1, x_2, \ldots, x_n \in (0, 1]$ such that $f(x_1, x_2, \ldots, x_n) = 0$ [24]. Conversely, we say f does not have zero divisors if $f(x_1, x_2, \ldots, x_n) = 0$,

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