Pattern Recognition Letters 38 (2014) 34–37

Contents lists available at ScienceDirect

Pattern Recognition Letters

journal homepage: www.elsevier.com/locate/patrec

Alhazen and the nearest neighbor rule

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ARTICLE INFO

Article history:
Received 25 May 2013
Available online 7 November 2013

Comminicated by S. Sarkar

Keywords:
Alhazen
Nearest neighbor classification
Visual perception
History of science

ABSTRACT

We show that a clear formulation of the nearest neighbor decision rule for pattern classification can be found, along with other remarkably modern ideas, in an influential medieval treatise on visual perception authored by Alhazen, one of the major figures in the so-called "Islamic golden age." To put the work in context we provide also a brief description of some of the salient points of Alhazen's theory.

1. Introduction

This is how Thomas Cover, many years later, detailed the origins of the nearest neighbor (NN) decision rule for pattern classification:

> Early in 1966 when I first began teaching at Stanford, a student, Peter Hart, walked into my office with an interesting problem. He said that Charles Cole and he were using a pattern classification scheme which, for lack of a better word, they described as the nearest neighbor procedure. This scheme assigned to an as yet unclassified observation the classification of the nearest neighbor. Were there any good theoretical properties of this procedure? (Cover, 1982)

Eventually, after several afternoon meetings, they were able to prove that the probability of error of this simple classification rule is bounded above by twice the Bayes minimum probability of error, and published one of the most influential papers in pattern recognition[1] (Cover and Hart, 1967). Nearest neighbor search, sometimes referred to as the "post office problem" (Knuth, 1973), arises also as a fundamental problem in a variety of computer science areas ranging from information retrieval to computational geometry and coding theory—see (Papadopoulos and Manolopoulos, 2005) for a database perspective.

Although, as pointed out by Cover himself, the basic *principle* motivating the NN rule, namely that "things that look alike must be alike," is lost in the mists of time (traces of which can in fact be found already in the earliest extant philosophical fragments), we show in this paper that a neat, explicit formulation of the NN rule *as a classification procedure* can be found in a little known part of an otherwise enormously influential medieval treatise which paved the way for the later development of modern visual perception theories.

The author of this treatise was Abu Ali al-Hasan ibn al-Hasan ibn al-Haytham, better known in the West as Alhazen, who ranks among the most prominent figures in medieval Islamic science. He flourished in Egypt in the early eleventh century and wrote extensively on various topics including physics, astronomy, and mathematics. The most influential of his writings, however, is undoubtedly the *Kitab al-Manazir* (or "Book of Optics"), which was written probably around the 1030's and translated into Latin in the late twelfth or early thirteenth century under the title *De Aspectibus* or *Perspectiva*.

Influenced by Ptolemy's optical theory, Alhazen's achievements in the field of visual perception are astonishing and were until recently neglected by the modern Western tradition. He anticipated by centuries many fundamental ideas that are still alive today such as, for example, Helmholtz's principle of unconscious inference, the apparent distance account of the moon illusion, the role of eye movements in visual perception, and he is also regarded as a precursor of the scientific method. Howard (1996) counted as many as eleven such anticipations and provided a detailed account of their development.

In this paper, we add one more item to the list. In fact, as it turns out, an important, though neglected, component of Alhazen's theory was a simple classification mechanism which is essentially identical to Cover and Hart's NN rule. We take this opportunity to make some of Alhazen's remarkable ideas on visual recognition known within the pattern recognition community. In particular, in Section 2 we provide a brief summary of Alhazen's optical theory, while in Section 3 we describe the psychological component of his account (where, as we shall see, we encounter the NN classification

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[1] In 1998 the paper received a Golden Jubilee Paper Award from the IEEE Information Theory Society.

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http://dx.doi.org/10.1016/j.patrec.2013.10.022

rule). Finally, in Section 4, we offer some speculations concerning the originality of Alhazen's proposal.

2. Alhazen's optical model

Of the seven books which compose Alhazen's treatise, the one which had the most durable impact on subsequent thinkers was mainly the first, which deals with such topics as the propagation of light, the anatomy of the eye, the visual pathways, etc., and expounds a systematic optical theory that was to put an end to long-lasting controversies. Indeed, for several centuries various rival doctrines of light and vision coexisted side by side, a state of affairs which motivated Kuhn (1970) to use this as an illustrative example of what he called a "pre-paradigmatic" stage of a scientific theory. Followers of the Euclidean and Ptolemaic traditions defended an extramission doctrine which postulates the existence of visual rays emanating from one's eye; the adherents of the atomistic school maintained an opposite view according to which thin replicas of the visible bodies emanate continually in all directions to enter the observer's eye. And there were of course variations of the basic themes such as Plato's combined extramission-intromission theory, and Aristotle's intromission version which insisted on the changes produced by the visible bodies on a transparent medium (Ronchi, 1952; Lindberg, 1976; Wade, 1998).

Alhazen would have none of this. Using a variety of ingenious arguments, including the phenomenon that today we call afterimage, he commences the first book of his *De Aspectibus* by providing compelling evidence against the extramission theory. He therefore sided with the intromission camp, but the theory he developed was radically different from the previous ones.

As pointed out by Lindberg (1976), Alhazen drew together the three optical traditions of his time, the mathematical, the anatomical and the physical, thereby creating a single comprehensive theory. The most revolutionary part of his theory, however, is neither the anatomical (which, as he himself admitted, was essentially that of Galen[2]), nor the mathematical. Instead, it was his physical explanations of optical phenomena, and in particular his punctiform analysis of visible bodies, that had the most lasting impact. Roughly, according to Alhazen's theory the surfaces of visible objects are thought of as composed of minute patches (points) which, when illuminated by a visual source, radiate their image rectilinearly in all directions. Some of these radiations enter the observer's eye through the pupil, wherever they meet, thereby giving rise to a series of processes which culminate in the experience of visual perception—see (Russell, 1996) for a more accurate description of the image formation process. Note that this intromissionist account of optical phenomena differs markedly from the preceding ones because, contrary to tradition, here it is not the body, taken as a whole, but each of its constituent points from which visual form issues. This might sound like today's textbook explanation, but we have to wait until Kepler to get the first truly modern optical theory.

The main problem Alhazen had to face was of course to explain how this collection of tiny images are reassembled into the eye to get a coherent picture of external objects. Note that, following Galen, he believed that the sensitive organ within the eye was not the retina but the crystalline, which was thought to be in a more central position than it actually is. Alhazen understood image formation in the eye in terms of a pinhole camera, of which he is credited to be the inventor, but realized that the pupil is too large to allow the eye to work precisely that way, as visual rays would intermingle and confusion would therefore arise. Since at the time of his writing he lacked a clear understanding of how lens work, a topographic, point-to-point correspondence between the visual field and the crystalline was difficult to establish. Alhazen's ingenious solution to this problem is recognized to be one of his greatest achievements in vision. He understood the importance of the role played by refraction in image formation and contended that, of all the visual rays emanating from a single point of a body's surface, only the one which hits the cornea perpendicularly, and hence is not bent by refraction, contributes to the formation of the image in the interior of the eye. In fact, Alhazen believed that refracted rays must lose their power to stimulate the sensitive organ, a principle that is reminiscent of what is known today as the Stiles-Crawford effect.

Alhazen's ideas were largely ignored for about 250 years, but they eventually came to dominate Western optical thought up to the beginning of the seventeenth century, deeply influencing scientists and philosophers such as Roger Bacon, John Pecham, and Witelo, only to fall again into oblivion until its recent rediscovery (Lindberg, 1976; Wade, 1998).

3. Beyond optics: visual recognition as nearest neighbor search

Some six centuries after Alhazen, and indirectly influenced by him, Kepler provided the first correct explanations of the mechanisms underlying image formation in the eye, and claimed he had essentially solved the problem of vision (or, at least, that it was not his business to investigate further). As he put it: "I say that vision occurs when the image of the whole hemisphere of the world that is before the eye [...] is fixed on the reddish white concave surface of the retina" (Lindberg, 1976, p. 203).

Following a tradition which goes back to Aristotle and to some extent to Galen, Alhazen held a more sophisticated view. In fact, after explaining the basic mechanisms of image formation, towards the end of Book I he hastens to say:

> in terms of naked sensation, sight perceives only the light and color that are in the visible object. The remaining characteristics of visible objects that sight perceive, e.g., shape, size, and the like, are perceived by sight not through naked sensation but through reason and defining features. And we shall show this later in the second book [...] [3] (Smith, 2001, p. 374).

It is indeed the second book of his treatise, which has typically been neglected by ancient and modern scholars alike (Sabra, 1978; Howard, 1996), which concerns us the most as it is here that we find a detailed exposition of the NN rule for classification, together with a series of remarkably modern observations about the nature of visual cognition. The account provided in this book can be considered in all regards as the first modern psychological theory of visual perception.

In Alhazen's theory, vision unfolds basically in three stages of increasing abstraction and complexity, starting from the passive registration of light and color in the eye. As he put it:

> of [all] the characteristics that are perceived by visual sensation, some are perceived through brute sensation, some through recognition, and some through judgment and differentiation. (Smith, 2001, p. 433)

Of these stages, it is the second which concerns us here as it is the faculty of recognition, and nothing else, which is responsible for visual categorization:

> Sight also perceives many things by means of recognition, so it recognizes that a human is a human, that a horse is a horse, and that

[2] In fact, he did not have a chance to do anatomical observations as Muslim's tradition forbade the dissection of the human body.

[3] All quotations from Alhazen's treatise are taken from a recent English translation of the Latin edition *De Aspectibus*, which is more readily accessible to the non-specialist than the classical Arabic-to-English counterpart (Sabra, 1989).