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Robust non-rigid point registration based on feature-dependant finite mixture model

Qiang Sang [a,*], Jian-Zhou Zhang [a], Zeyun Yu [b]

[a] College of Computer Science, Sichuan University, Chengdu 610065, China
[b] Department of Computer Science, University of Wisconsin-Milwaukee, Milwaukee, WI 53211, USA

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ABSTRACT

In previous works on point registration based on finite mixture model, the correspondence probability is often determined by exploiting global relationship in the point set instead of considering the local point distribution. That results in a simplified registration model. In this paper a feature-dependant finite mixture model (FDMM) is proposed. In particular, an improved descriptor is introduced to describe the local feature of a point. Consequently, a priori density function is formulated for the mixture weights. The unknown parameters of FDMM are computed by maximizing a posteriori (MAP) estimation. Moreover, a bidirectional expectation–maximization (EM) process is introduced to update both point sets in contrast to traditional methods. The performance of our method is demonstrated and validated with carefully designed synthetic data and real data, showing that the proposed method can improve the robustness and accuracy as compared to the traditional registration techniques.

1. Introduction

The registration between two point sets is a fundamental problem in computer vision, medical image analysis and pattern recognition. In fact, it is the key component in many applications such as motion tracking, shape matching, context-based image retrieval and image registration. The goal of registration is to find meaningful correspondence between two different point sets and determine the transformation that maps one set to the other. These point sets are often extracted from other types of data such as images representing feature points. Because contour extraction and image segmentation are still an open problem, these point sets extracted are often corrupted by a lot of outliers which seriously degrade the performance of registration. How to improve the robustness is of fundamental importance in point set registration.

Generally speaking, there are roughly three categories of methods on point set registration: the iterative closest point (ICP) algorithm (Besl and Mckay, 1992; Zhang, 1994) and its extensions (Fitzgibbon, 2001; Chetverikov et al., 2005), soft assignment methods (Chui and Rangarajan, 2003; Yang, 2011) and probabilistic methods (Chui and Rangarajan, 2000; Myronenko and Song, 2010; Horaud et al., 2011; Jian and Vemuri, 2005).

The ICP introduced by Besl and Mckay (1992) and Zhang (1994), is one of the commonly used methods, which iteratively assigns correspondence based on the Euclidean distance and finds the least squares based transformation related to these point sets. Many variants of ICP have been proposed that modify the phases of the algorithm from the selection and matching of points to the minimization strategy. Fitzgibbon (2001) applies a robust loss function to the Euclidean distance and yields a non-linear version of ICP. Another method is to select trimmed subsets of points by repeating random sampling as the TriICP algorithm proposed by Chetverikov et al. (2005). Because ICP assigns the definite point-to-point correspondence in each iteration, it is easy to get stuck in local minima. Also, ICP is sensitive to initial locations of point sets, such that these point sets must be adequately close to each other, especially in non-rigid registration. So ICP is usually applied to rigid registration.

The robust point matching (RPM) (Chui and Rangarajan, 2003) improves the performance in contrast to the ICP by adopting soft assignment and deterministic annealing techniques. The RPM deals with outliers by adding one column and one row to similarity matrix. Several data points are allowed to be assigned to this extra column and, symmetrically, several model points may be assigned to this extra row. Therefore, the resulting algorithm provide one-to-one assignments for inliers and many-to-one assignments for outliers. Yang (2011) extends the original RPM by a double sided outlier handing approach. However, these approaches are not truly probabilistic. Although using similar EM, they do not strictly compute the posterior probability in the “E” step.

Recently some algorithms based on the finite mixture model (FMM) have been proposed. Chui and Rangarajan (2000) formulate feature registration problems as maximum likelihood estimation

* Corresponding author. Tel.: +86 18280235299; fax: +86 028 85413223.
E-mail addresses: sangqiang2000@gmail.com (Q. Sang), yuz@uwm.edu (Z. Yu).

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problems using mixture models. The approach is the embedding of the EM algorithm within a deterministic annealing scheme in order to directly control the fuzziness of the correspondences. Coherence point drift (CPD) (Myronenko and Song, 2010) defines a velocity function for the template point set, namely the centroid of the Gaussian mixture model (GMM), and iteratively calculates the unknown parameters in the GMM by EM. An expectation conditional maximization (ECM) algorithm for point registration called ECMPR is proposed by Horaud et al. (2011), which adopts the anisotropic covariance model and ECM to resolve the rigid and articulated point registration. In order to enhance robustness, these methods include an extra uniform component (Hennig and Coretto, 2008) in the mixture model. But this strategy cannot fit outliers in both point sets at the same time. Sanroma et al. (2012) propose a rigid point set registration method that uses neighboring relation and extends a non-rigid registration version. However, these algorithms simplify the posterior probability due to an improper a priori that assuming an average mixing weight. Another approach (Jian and Vemuri, 2005) is to model the two point sets by two GMM and estimate the model parameters by minimizing the dissimilarity. However, the outliers are still not explicitly modeled.

In this paper we propose to enhance the robustness and accuracy of point registration algorithm. The contributions of the paper are: (1) We propose a probabilistic registration model which combines global relationship with local feature of a point. (2) We introduce a improved local feature descriptor to measure the non-rigid deformation accurately. (3) In order to fit the outliers and take advantage of local feature of a point in two point sets at the same time, we adopt a bidirectional update processing.

The present paper is organized as follows. Section 2 gives the problem formulation. A description of previous probability work and their deficiency are presented in the next section. The proposed approach is developed in Section 4, followed by a number of experiments and validations using synthetic and real examples. Discuss and conclusions are drawn in Section 6.

2. Problem formulation

Given two point sets $X = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_i, \ldots, \vec{x}_N\}$ and $Y = \{\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_j, \ldots, \vec{y}_L\}$, where $\vec{x}_i$ and $\vec{y}_j$ are d-dimension vectors, N and L are the numbers of points in X and Y respectively. The goal of registration is.

- to calculate the transformation that maps one point set to the other (we denote the mapping by $g : \Re^d \rightarrow \Re^d$, $\vec{y}_j = g(\vec{x}_i, \vartheta_t)$, where ϑ_t denotes a set of unknown transformation parameters);
- to determine the correspondence between X and Y according to the above-mentioned transformation.

3. The classical mixture model for point registration

In the point registration based on classical mixture model (CMM) (McLachlan and Peel, 2000a), $\vec{x}_i (1 \leqslant i \leqslant N)$ denotes the observation at the ith point of one point set X and the other point set denotes L finite mixture model components whose density functions are $f_j(\vec{x}_i|\vec{\theta}^j)$. Let observation $\vec{x}_i$ be modeled as i.i.d. Now the joint conditional density of the observation is formed as

$$f_x(\vec{x}|\vec{\pi}, \vec{\theta}^1 \cdots \vec{\theta}^L) = \prod_{i=1}^{N} \sum_{j=1}^{L} \vec{\pi}_j f_j(\vec{x}_i|\vec{\theta}^j) \quad (1)$$

where $\vec{\pi}_j$ is the mixing weight and $\vec{\theta}^j$ is the parameter of the density function. In the case of GMM,

$$f_j(\vec{x}_i|\vec{\theta}^j) = \frac{1}{\sqrt{2\pi}\vec{\sigma}_j} \exp\left[-\frac{(\vec{x}_i - \vec{\mu}_j)^2}{2(\vec{\sigma}_j)^2}\right] \quad (2)$$

where $\vec{\mu}_j$ is the mean and $(\vec{\sigma}_j)^2$ is the variance. Here $\vec{\mu}_j$ denotes the updated position of GMM centroid, namely the point $\vec{y}_j$ in point set Y. The point $\vec{x}_i$ belongs to the jth class if $\vec{x}_i$ is the corresponding point of $\vec{y}_j$. Thus the question of point registration can be interpreted as clustering of point set. So the parameters of the component densities $(\vec{\pi}_j, \vec{\mu}_j, \vec{\sigma}_j)$ can be estimated by the iterative EM process. According to (McLachlan and Peel, 2000a):

$$\vec{y}_j^{(k+1)} = \vec{\mu}_j^{(k+1)} \Rightarrow g(\vec{x}_i, \vartheta_t) = \frac{1}{N\vec{\pi}_j^{(k+1)}} \sum_{i=1}^{N} \vec{w}_j^{i^{(k)}} \vec{x}_i \quad (3)$$

where k denotes the number of iterations in the EM algorithm and $\vec{w}_j^{i^{(k)}}$ is the posterior probability. When the CMM parameter converges, the transformation parameter ϑ_t will be obtained by (3).

In order to determine the correspondence, a vector $\vec{p}^i$ is introduced. Let the probability of $\vec{x}_i$ mapped to $\vec{y}_j$ be denoted by $\vec{p}_j^i$. If the point $\vec{x}_i$ is the corresponding point of $\vec{y}_j$ in point set Y then $\vec{p}_j^i = 1$ otherwise $\vec{p}_j^i = 0$, so that $Prob(\vec{p}_j^i = 1) = \vec{\pi}_j, \forall i$. Given the component density $f_j(\vec{x}_i|\vec{\theta}^j)$, the Bayes rule gives

$$Prob(\vec{p}_j^i = 1|\vec{x}_i, \vec{\theta}^j) = \frac{\vec{\pi}_j f_j(\vec{x}_i|\vec{\theta}^j)}{\sum_{l=1}^{L} \vec{\pi}_l f_l(\vec{x}_i|\vec{\theta}^l)} \quad (4)$$

When the parameters $\vec{\theta}^j$ of the density function are known, the Bayes classification rule could be used to find the correspondence of point $\vec{x}_i$ by solving

$$max_j Prob(\vec{p}_j^i = 1|\vec{x}_i, \vec{\theta}^j). \quad (5)$$

Although Myronenko and Song (2010) and Horaud et al. (2011) take advantage of GMM to register point sets, they just use the simplified version that assumes an improper a priori, namely $\vec{\pi}_j$ is a constant. So in the ''E'' step the posterior probability

$$\vec{w}_j^{i^{(k)}} = \frac{\vec{\pi}_j^{(i)} f_j(\vec{x}_i|\vec{\mu}_j^{(k)}, \vec{\sigma}_j^{(k)})}{\sum_{l=1}^{L} \vec{\pi}_l^{(i)} f_l(\vec{x}_i|\vec{\mu}_l^{(k)}, \vec{\sigma}_l^{(k)})} \quad (6)$$

would be simplified as

$$\vec{w}_j^{i^{(k)}} = \frac{f_j(\vec{x}_i|\vec{\mu}_j^{(k)}, \vec{\sigma}_j^{(k)})}{\sum_{l=1}^{L} f_l(\vec{x}_i|\vec{\mu}_l^{(k)}, \vec{\sigma}_l^{(k)})} \quad (7)$$

When there are not outliers among two point sets and the sizes of two point sets are equal$(N = L)$, the assumption is acceptable. Otherwise, the outliers also would be treated as the centroid of GMM and share the same weight with others. That hinders the EM process from converging correctly.

4. The feature-dependant mixture model for point registration

In order to solve the problem mentioned above, we incorporate a new parameter $\vec{p}_j^i$ in finite mixture model as Sanjay-Gopal and Hebert (1998). The $\vec{p}_j^i$ denotes the probability of the ith point belonging to the jth component and $0 \leqslant \vec{p}_j^i \leqslant 1$, $\sum_j \vec{p}_j^i = 1, \forall i$. Thus (1) is redefined as follows:

$$f_x(\vec{x}|\vec{p}^1 \cdots \vec{p}^N, \vec{\theta}^1 \cdots \vec{\theta}^L) = \prod_{i=1}^{N} \sum_{j=1}^{L} \vec{p}_j^i f_j(\vec{x}_i|\vec{\theta}^j) \quad (8)$$

When $\vec{p}_j^i = \vec{\pi}_j$, it is a special case as used in (Myronenko and Song, 2010; Horaud et al., 2011).

As defined in (Sanjay-Gopal and Hebert, 1998; Titterington et al., 1995), the complete data $\vec{t}(\vec{t}^T \equiv (\vec{x}^T, \vec{z}^{1^T}, \ldots, \vec{z}^{N^T}))$ with superscript T denoting vector transpose is used. Here $\vec{z}^i (i = 1, 2, \ldots, N)$ denote $L \times 1$ random indicator vectors, each of which takes a value from the set of vectors $\zeta = \{\vec{e}^j; \vec{e}^j_{l=j} = 1, \vec{e}^j_{l \neq j} = 0, 1 \leqslant j, l \leqslant L\}$, where $\vec{e}^j$ is a $L \times 1$ vector in which one of vector components is one and the others are zero. Thus $\vec{z}_j^i$ is a discrete random variable with